AP CALCULUS STUDY GUIDE

Use of Graphing Calculators

- Graph/Adjust Windows/Create a Table of Values for a Function
- Calculate Minimum/Maximum Value of a Function
- Calculate the Intersection of Two Functions
- Find the Zeros of a Function
- Solve Equations
- Store Values in Memory
- Calculate Values of Derivatives and Integrals (in numerical terms)

Existence of a Limit

$\lim_{x \to c} f(x) = L$ if and only if

$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$

L'Hopital's Rule

If $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0} \left(or \ \frac{\infty}{\infty} \right)$,

then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$

Continuity of a Function

$f(c)$ is defined

$\lim_{x \to c} f(x)$ exists, and

$\lim_{x \to c} f(x) = f(c)$

Limit Definition of Derivative

$$slope = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Intermediate Value Theorem

If $f(x)$ is continuous on [a, b] and $f(a) < k < f(b)$, then there exists at least one value c in [a, b] such that $f(c) = k$.

Rolle's Theorem

If $f(x)$ is continuous on [a, b], differentiable on (a, b), and $f(a) = f(b)$, then there exists at least one value c in (a, b) such that $f'(c) = 0$ (*so* $f(x)$ *has a horizontal tangent line*).

Mean Value Theorem

If $f(x)$ is continuous on [a, b] and differentiable on (a, b), then there exists a value of c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

(***so the instantaneous rate of change is equal to the average rate of change***).

Differentiation Rules

Power: $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Product:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$$

Quotient:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Chain: $\frac{d}{dx}\left[f(u)\right] = f'(u)\frac{du}{dx}$

Basic Derivatives

$\frac{d}{dx}[c] = 0$

$\frac{d}{dx}[x] = 1$

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$

$\frac{d}{dx}\left[e^x\right] = e^x$

Other Derivatives

$\frac{d}{dx}\left[\arcsin u\right] = \frac{u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}\left[\arccos u\right] = \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}\left[\arctan u\right] = \frac{u'}{1+u^2}$

Graphical Analysis

critical numbers: $f'(x) = 0$ or is undefined (possible min/max values)

$f(x)$ is increasing if $f'(x) > 0$ and is decreasing if $f'(x) < 0$

$f(x)$ is concave up if $f''(x) > 0$ and is concave down if $f''(x) < 0$

$f(x)$ has a point of inflection where its concavity changes, or where $f''(x)$ changes sign

$f(x)$ has a relative/local minimum where $f'(x)$ changes from - to + and a relative/local maximum where $f'(x)$ changes from + to -

$f(x)$ has a relative/local minimum where $f''(x) > 0$ and a relative/local maximum where $f''(x) < 0$

$f(x)$ has an absolute/global minimum and maximum on [a, b] at the smallest and largest values of $f(x)$ after checking all critical numbers and endpoints

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Particle Motion

$velocity = \frac{d}{dx}[position]$

$acceleration = \frac{d}{dx}[velocity]$

speed= $|velocity|$

total distance travelled= $\int |velocity|$

a particle moves right when velocity>0, left when velocity<0

a particle speeds up when velocity and acceleration have the same sign, slows down when velocity and acceleration have opposite signs

Definite Integrals

If $f(x)$ >0 for all x on (a, b), then the area under the curve of $f(x)$ is given by

$$\int_a^b f(x)dx$$

Fundamental Theorem of Calculus

$\int_a^b f(x)dx = F(b) - F(a)$, where F(x) is the antiderivative of $f(x)$

($\int_a^b f'(x)dx = f(b) - f(a)$ is equivalent)

Second Fundamental Theorem of Calculus

$\frac{d}{dx}\int_a^x f(x)dx = f(x)$ (or $\frac{d}{dx}\int_a^{g(x)} f(x)dx = f(g(x))g'(x)$ with chain rule applied)

Riemann Sums

For each subinterval find the area of the rectangle, then add to find the total approximation of the integral value.

Rectangle areas are found by

left Riemann Sum: (interval length)x(f(left endpoint))

right Riemann Sum: (interval length)x(f(right endpoint))

midpoint Riemann Sum: (interval length)x(f(midpoint))

Trapezoidal Sum

Find the area of each trapezoid with (interval length)/2 x [f(left endpoint)+f(right endpoint)]

Integration Rules

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int k\,dx = kx + C$$

$$\int f(u)\frac{du}{dx} = F(u) + C$$

$$\int \frac{1}{x}dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc\cot dx = -\csc x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$$

$$\int \frac{1}{1+x^2}dx = \arctan x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}}dx = arc\sec x + C$$

Average Value of a Function

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Exponential Growth and Decay

When the rate of change of the variable is proportional to the variable itself, the situation is modeled by the equation $y = Ce^{kt}$, where C is the initial value of y, k is the proportionality constant or rate of growth/decay, and t is time.

Limits at Infinity

$\lim_{x\to\infty}\frac{ax^n + ...}{bx^m + ...} = 0$, if n < m

$\lim_{x\to\infty}\frac{ax^n + ...}{bx^m + ...} = \frac{a}{b}$, if n = m

$\lim_{x\to\infty}\frac{ax^n + ...}{bx^m + ...}$ does not exist, if n > m

Derivative of an Inverse Function

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

Volumes of Revolution

Disk Method: $V = \pi\int_a^b [R(x)]^2\,dx$

Washer Method:

$$V = \pi\int_a^b (\left[R(x)\right]^2 - \left[r(x)\right]^2)dx$$

Cross-Sectional Volume

$V = \int_a^b A(x)dx$, where A(x) represents the area of each cross section

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