Slope Fields & Euler's Method (5.1)

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I. General & Particular Solutions

A function y=f(x) is a <u>solution</u> to a differential equation if the equation is true when y and its derivatives are replaced by f(x) and its derivatives.

The general solution of a first-order differential equation represents a family of solution curves that include a constant C. Given initial conditions, we can obtain a particular solution of the differential equation.

Ex. 1: Find a general solution of the differential equation

$$\frac{dy}{dx} = -5xe^{3x^2}$$

II. Slope Fields

Slope fields, or direction fields, can be used to analyze the solution to a differential equation that cannot be found analytically. Given a differential equation of the form $y'=F(x, y)$ we can plug in any point (x, y) to find the slope y' of the solution of the equation at that point. The sketch of short line segments that show the slope at each point is called a slope field.

Ex 2: Consider the differential equation $\frac{dy}{dx} = x^2 + 1$, where $x \neq 0$.

(a) Sketch a slope field for the given differential equation for $-2<x<2$, $x \neq 0$, and $-1<y<1$.

(b) Find the particular solution y=f(x) to the differential equation with initial condition f(-1)=1 and state its domain. Sketch this solution on top of your slope field.

Euler's Method for Approximating the Particular Solution to a Differential Equation

This method uses the tangent line, given the initial condition (x_0, y_0) and slope at that point, to get the value of an approximation for another point on the graph of y. Each successive point with be an arbitrary distance, h, away from the starting point.

This will give a table of values and thus an approximate graph for the solution y.

Ex. 3: Use Euler's Method to approximate the particular solution of the differential equation $y' = x + y$ passing through the point (0, 2). Use 10 steps of size h=0.05.