Differential Equations: Growth & Decay (5.2)

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I. Differential Equations

To solve a differential equation in terms of x and y, separate the variables.

II. Exponential Growth & Decay Models

In applications, it is often true that

$$\frac{dy}{dt} = ky$$

the rate of change of y with respect to time

is

proportional to y

Thm. 6.1: Exponential Growth & Decay Model: If y is a differentiable function of t such that $y>0$ and $y'=ky$, for some constant k, then $y=Ce^{kt}$.
C is the initial value of y, and k is the proportionality constant. Exponential growth occurs when $k>0$, and exponential decay occurs when $k<0$.

*Recall that for radioactive decay applications, a half-life of 20 years will allow you to find the decay model $y=Ce^{kt}$ with the equation $1/2=e^{20k}$.

*Also, for compound interest applications, the models for growth are

$$A = Pe^{rt}$$ when interest is compounded continuously

and $$A = P\left(1+\frac{r}{n}\right)^{nt}$$ when compounded n times per year.

P=principal amount, r=rate, t=time (in years)

Ex. 1: One hundred elk are introduced into a game preserve. Assume that the elk population is increasing exponentially such that the growth rate is 8%.

a) Find a differential equation for the growth model for the elk.
b) Find an exponential growth model for the elk.
c) Estimate the number of elk alive after 10 years.
d) Estimate the number of years when the population will be doubled.

Ex. 2: Scientists who do carbon-14 dating use 5700 years for its half-life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

III. Newton's Law of Cooling Model

*The rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Thus, if y represents the temperature of the object in a room kept at 60° , then

$$\frac{dy}{dx} = k(y - 60)$$

Ex. 3: If a cup of hot coffee cools from $120°F$ to $60°F$ in 40 minutes in a room of temperature

$35°F$, use Newton's law of cooling to find the temperature of the coffee after 100 minutes.