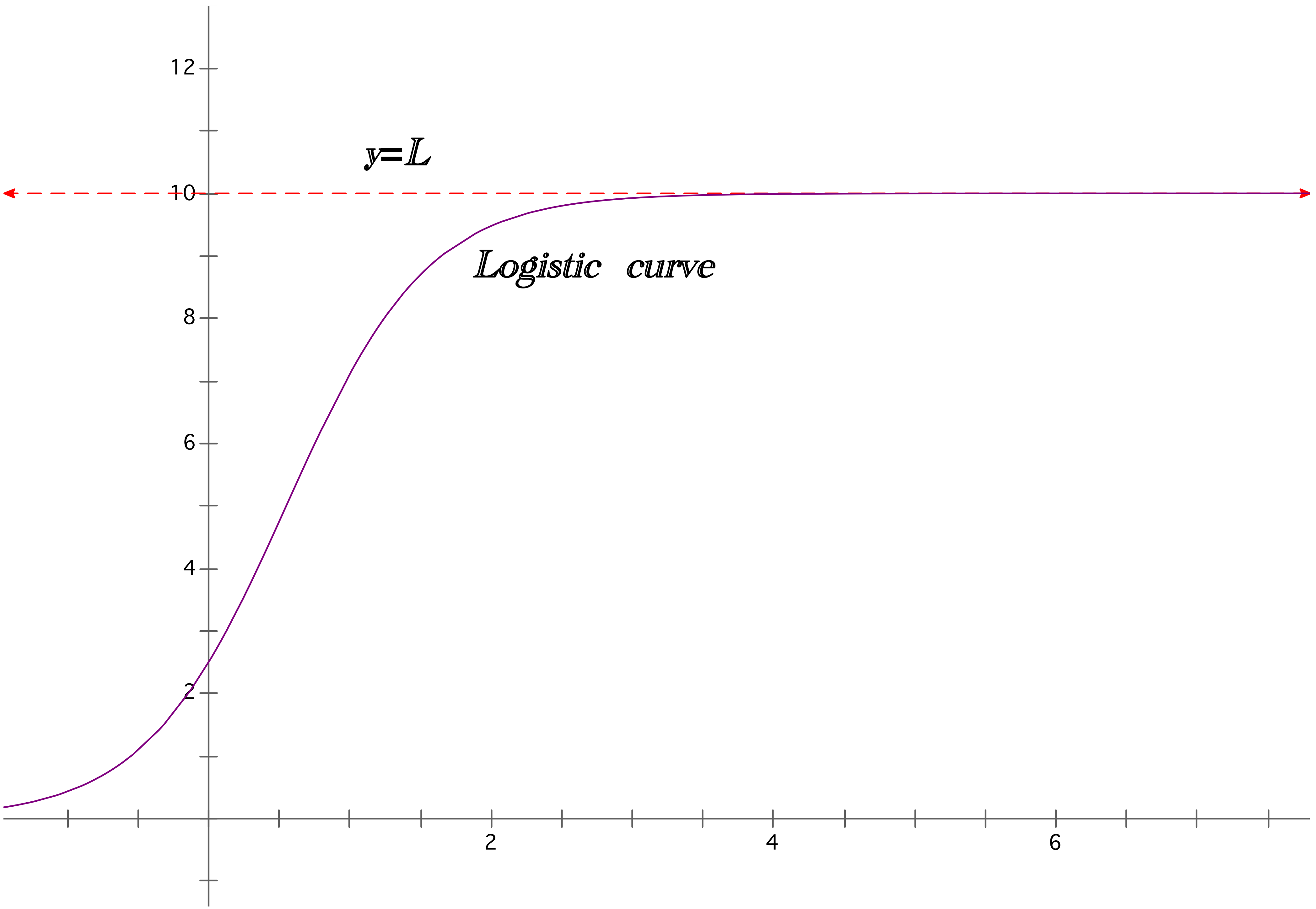
The Logistic Equation (5.4)

December 21st, 2018

The Logistic Differential Equation

*Unlike a basic exponential growth model, most populations have some upper limit L past which growth cannot occur. These can be modeled by the logistic differential equation, $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$, where k and L are positive constants and L is called the <u>carrying capacity,</u> which is the maximum population y(t) that can be supported or sustained as t increases. If $\frac{dy}{dt} > 0$, the population increases, whereas if $\frac{dy}{dt} < 0$, the population decreases.



As $t \to \infty$, $y \to L$

Ex: The population of Nowheresville is 20,000 people. After 4 years, the population is 22,500 people. It is believed that Nowheresville can only sustain 100,000 people.

a) Write a logistic differential equation that models the population of Nowheresville.

b) Find a general solution to the logistic differential equation $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$ that can be used for all logistic models.

c) Find the logistic model for Nowheresville.

d) Use the model to estimate the population after 20 years.

e) Find the limit of the model as $t \to \infty$