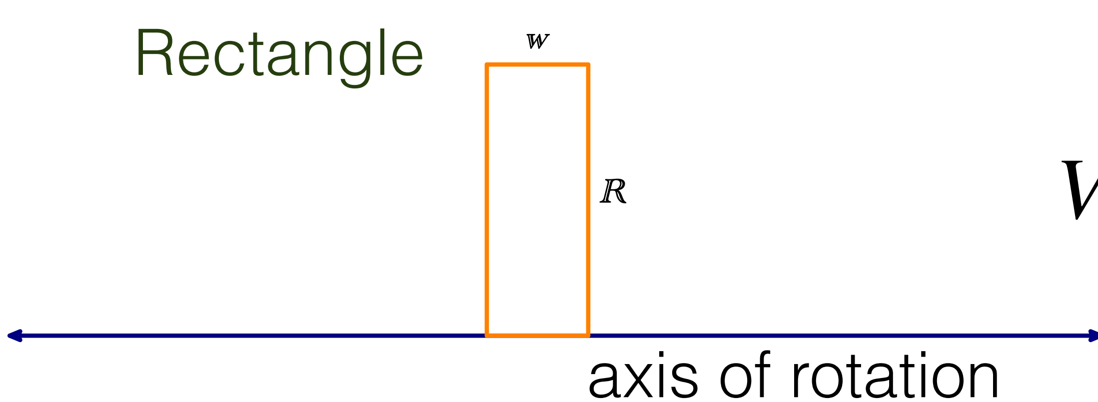
# The Disk Method (6.2)

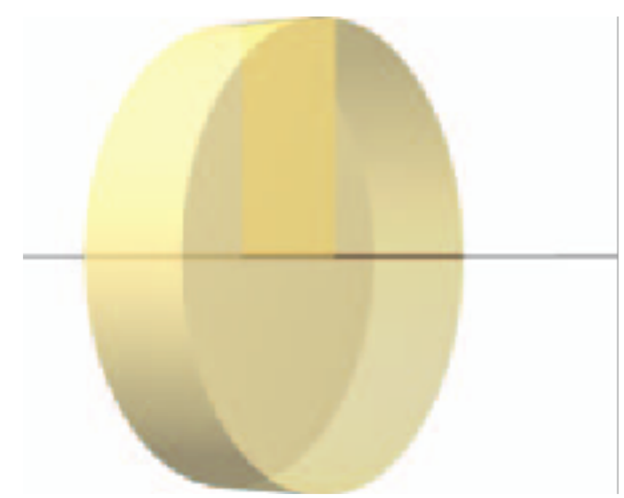
January 29th, 2019

# I. The Disk Method

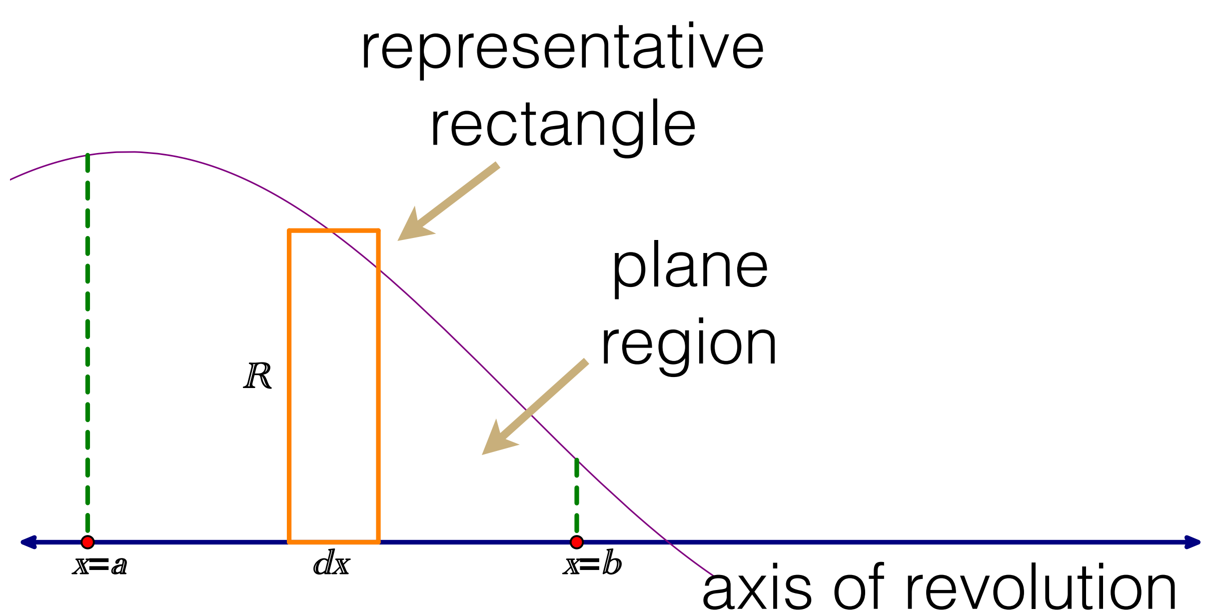
Def. If a region in the coordinate plane is revolved about a line, called the axis of revolution, the resulting solid is a solid of revolution. A disk (or right circular cylinder) is the result of revolving a rectangle about an axis adjacent to one of the rectangle's sides.

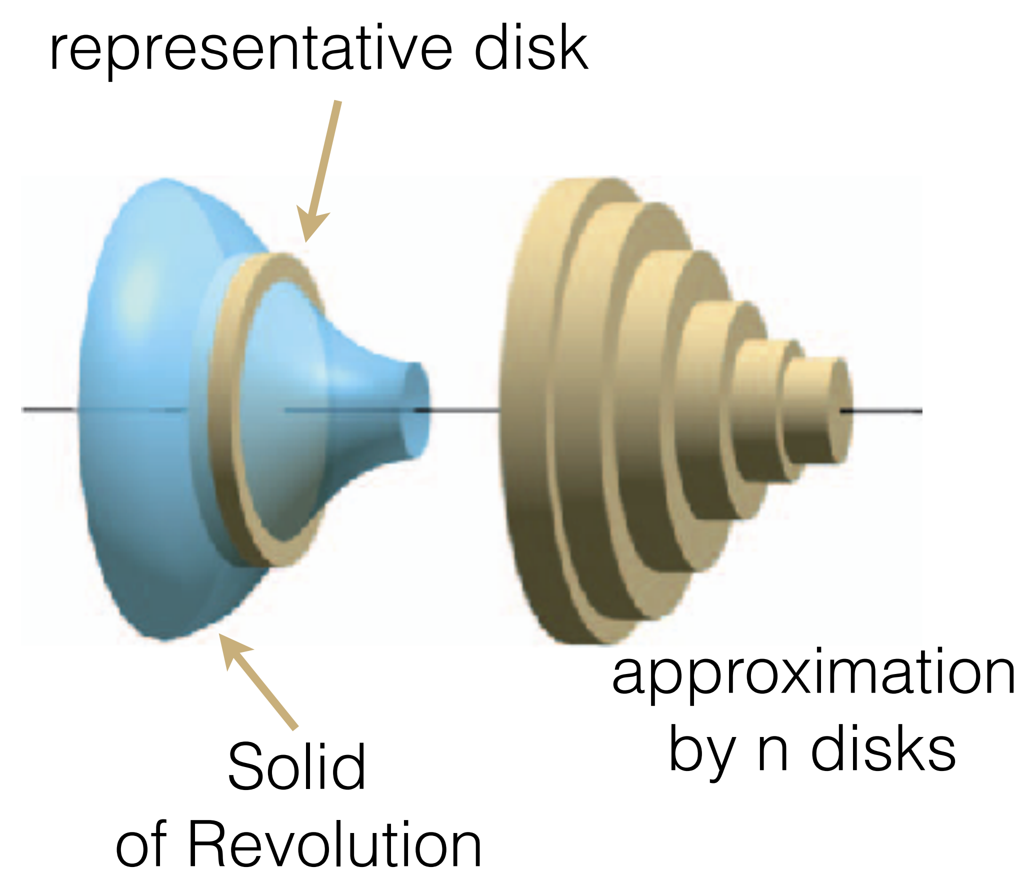



$$Volume = \pi R^2 w$$



*We can apply this concept to finding the volume of a solid of revolution formed by rotating **any** given region in a plane by using an integral (since an integral represents the limit of an infinite number of rectangles that is used to calculate the area of the region).

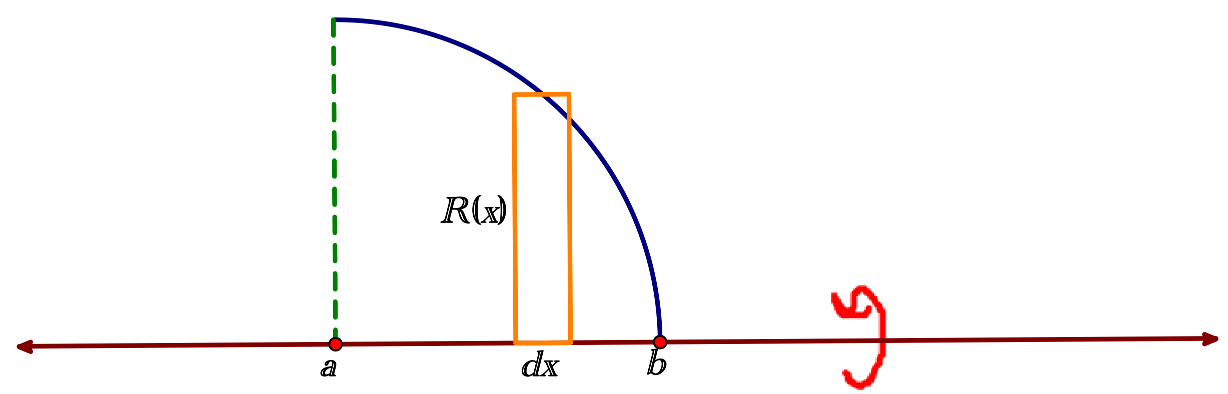


The Disk Method: To find the volume of a solid of revolution with the disk method, use one of the following:

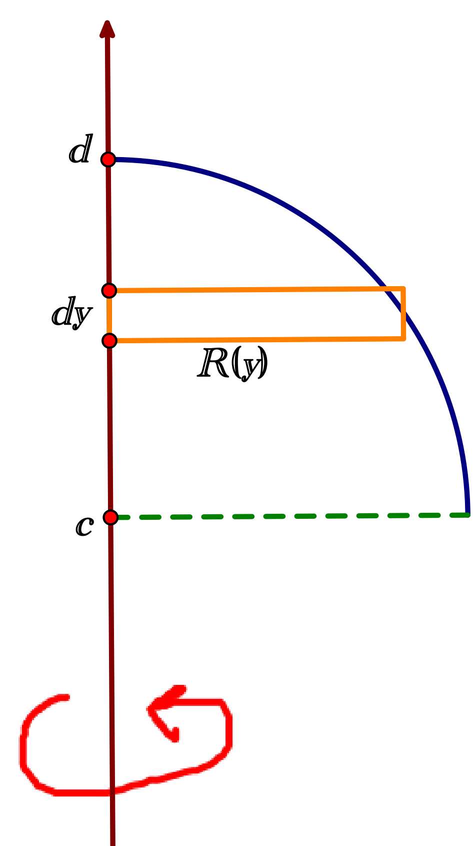
Horizontal axis of revolution:

$$V = \pi \int_a^b \left[ R(x) \right]^2 dx$$



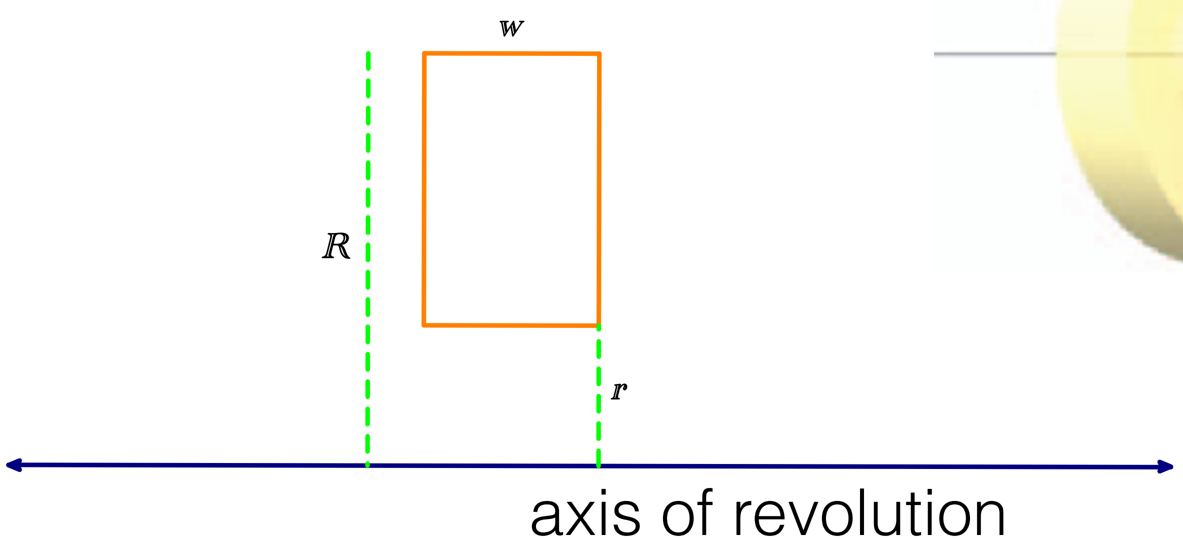

Vertical axis of revolution:

$$V = \pi \int_c^d \left[ R(y) \right]^2 dy$$

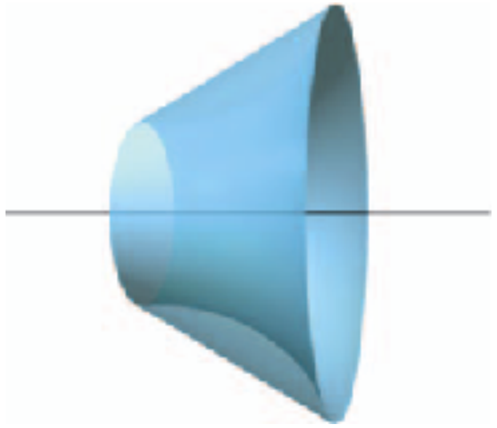


# II. The Washer Method

*If revolving a region results in a solid of revolution with a hole, we must extend the disk method with the washer method.




Washer



Solid of Revolution with hole

The Washer Method: To find the volume of a solid of revolution generated by the revolution of a region bounded by an outer radius R(x) and an inner radius r(x), use

$$V = \pi \int_a^b \left( [R(x)]^2 - [r(x)]^2 \right) dx$$

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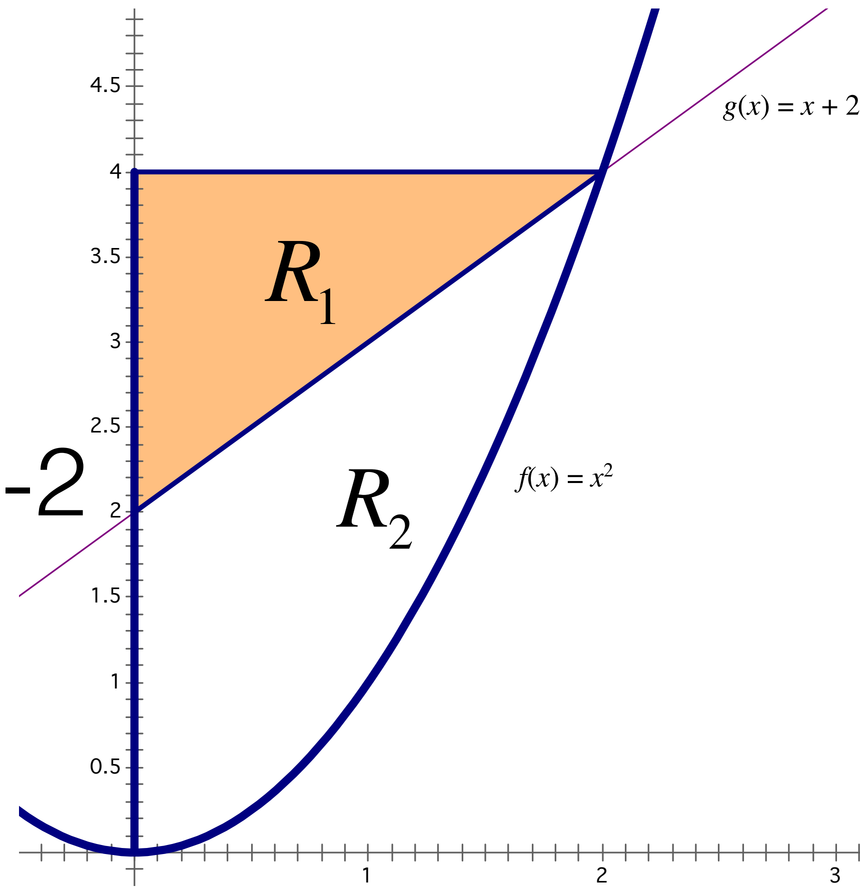
*Recall that the disk method is just the simpler version of the washer method, since $r(x)=0$ as a result of the region being bound by the axis of revolution itself.

Ex. 1: Find the volume of the solid generated by revolving the specified region about the given line of revolution.

a) $R_2$ rotated about the x-axis
b) $R_1$ and $R_2$ rotated about the y-axis
c) $R_2$ rotated about the line y=4
d) $R_1$ and $R_2$ rotated about the line x=-2
e) $R_2$ rotated about the line x=2



# Cross-Sectional Volume

*Recall that cross-sectional volume involves using the 2-dimensional area formed by graphs, then building up cross-sections of geometric shapes to form a 3-dimensional object. We then find the volume by setting up an integral involving the appropriate geometric formula in which the original function equations give us a necessary length that is plugged into the formula.

# Volumes of Solids with Known Cross Sections

1. For cross sections of area A(x) taken perpendicular to the x-axis,

$$\text{Volume} = \int_a^b A(x)\,dx$$

2. For cross sections of area A(y) taken perpendicular to the y-axis,

$$\text{Volume} = \int_c^d A(y)\,dy$$

Ex. 2: Sketch and shade the region bound by the graphs of $f(x)=x$ and $g(x)=\frac{x^2}{2}-3$. Use this graph as the base of a solid and find the volume of the solid that results from the given cross sections that are perpendicular to the specified axis.

a) cross-sectional semicircles that are perpendicular to the x-axis

b) cross-sectional isosceles right triangles with a leg lying perpendicular to the x-axis

c) cross-sectional squares that are perpendicular to the y-axis