Integration by Parts (7.2)

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*<u>Thm. 8.1: Integration by Parts:</u> If u and v are functions of x and have continuous derivatives, then

$$\int u\,dv = uv - \int v\,du \quad .$$

*Let dv be the most complicated part of the integrand that fits a basic integration rule. Then u is the remaining factors of the integrand.

-or-

*Let u be the part of the integrand whose derivative is simpler than u. Then dv is the remaining factors of the integrand.

Proof of Thm 8.1: Integration by Parts Using the Product Rule for Differentiation

Ex. 1: Find $\int xe^{2x}\,dx$

You Try: Find $\int \frac{x}{e^{3x}} dx$

Ex. 2: Find $\int \frac{\ln x}{x^4} dx$

You Try: Find $\int x^6 \ln(2x)\,dx$

**You can repeat integration by parts, if necessary, to get the integrand progressively simpler.

Ex. 3: Find $\int x^2 \cos x \, dx$

You Try: Find $\int x^3 e^{3x}\, dx$

Summary: Common Integrals Using Integration by Parts:

1. For integrals of the form

$$\int x^n e^{ax}\,dx, \int x^n \sin ax\,dx, \text{ or } \int x^n \cos ax\,dx$$

let $u = x^n$ and let $dv = e^{ax}dx, \sin axdx, \text{ or } \cos axdx.$

2. For integrals of the form

$$\int x^n \ln x\,dx, \int x^n \arcsin ax\,dx, \text{ or } \int x^n \arctan ax\,dx$$

let $u = \ln x, \arcsin ax, \text{ or } \arctan ax$ and let $dv = x^n dx.$

3. For integrals of the form

$$\int e^{ax} \sin bx\,dx \text{ or } \int e^{ax} \cos bx\,dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}dx.$