Indeterminate Form & L’Hopital’s Rule (7.7)

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*When evaluating limits, indeterminate form refers to limits that result in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$. In this form we cannot guarantee the existence or non-existence of the limit

Thm. 7.3: The Extended Mean Value Theorem: If *f* and *g* are differentiable on the open interval (a, b) and continuous on the closed interval [a, b] such that $g'(x) \neq 0$ for any *x* in (a, b), then there exists a point *c* in (a, b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

*The Extended Mean Value Theorem (used within the context of a limit), leads us to L'Hopital's Rule.

Thm. 7.4: L'Hopital's Rule: Let *f* and *g* be functions that are differentiable on the open interval (a, b) containing *c*, except possibly at *c* itself. Assume $g'(x) \neq 0$ for all *x* in (a, b), except possibly at *c* itself. If the limit of *f(x)/g(x)* as *x* approaches *c* produces an indeterminate form, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Provided the limit on the right exists (or is infinite).

Ex. 1: Evaluate each of the following limits. Use L'Hopital's Rule when applicable.

$$a)\ \lim_{x\to 0}\frac{1-\cos x}{x}$$

$$b)\ \lim_{x\to 0}\frac{\tan x}{x}$$

$$c)\ \lim_{x\to \infty}\frac{e^x-1}{x}$$

$$d)\ \lim_{x\to -\infty}\frac{3x^3-x+1}{2x^2}$$

$$e)\ \lim_{x\to 0}\frac{2x^2-3x}{3x^2-2x-1}$$

$$f)\ \lim_{x\to \infty}\frac{x^2}{\ln x}$$

***L'Hopital's Rule can be repeated if the limit continues to result in indeterminate form.

Ex. 2: Evaluate each of the following limits.

$$a)\ \lim_{x \to -\infty} \frac{x^3}{e^{-x}}$$

$$b)\ \lim_{x \to \infty} \frac{(\ln x)^2}{x^2}$$

*There are other indeterminate forms of limits that may be able to be written in a form that have L'Hopital's Rule applied. These include $0 \cdot \infty$, 1^{∞}, 0^{0}, and $\infty - \infty$.

Ex. 3: Evaluate each of the following by manipulating the expression into a rational function that L'Hopital's Rule can be applied to.

$a)\ \lim_{x \to \infty} x^{3/2} e^{-x}$

$b)\ \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}$

$c)\ \lim_{x \to 0^{+}} (\sin x)^{x}$

$d)\ \lim_{x \to 1^{+}} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$