Improper Integrals(8.8)

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*Improper integrals are integrals that either:

Have an infinite limit of integration, or

Have a finite number of infinite discontinuities.

**Note that the definition of the definite integral and the Fundamental Theorem of Calculus do not work under these circumstances.

Definition of Improper Integrals with Infinite Integration Limits:

1. If *f* is continuous on the interval $[a,\infty)$, then

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx.$$

the improper integral converges if the limit exists

2. If *f* is continuous on the interval $(-\infty,b]$, then

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx.$$

the improper integral converges if the limit exists

3. If *f* is continuous on the interval $(-\infty,\infty)$, then

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx.$$

the improper integral on the left diverges if either of the two integrals on the right diverges

where *c* is any real number.

Ex. 1: Evaluate each improper integral.

$$a)\int_{1}^{\infty} \frac{5}{x^3}\,dx$$

$$b)\int_{-\infty}^{0} e^{2x}\,dx$$

$$c)\int_{-\infty}^{\infty} \frac{4x}{1+x^4}\,dx$$

$$d)\int_{0}^{\infty} (e^x+1)\,dx$$

Definition of Improper Integrals with Infinite Discontinuities:

1. If f is continuous on the interval [a, b) and has an infinite discontinuity at b, then

$$\int_a^b f(x)dx = \lim_{c \to b^-} \int_a^c f(x)dx.$$

the improper integral converges if the limit exists, otherwise it diverges

2. If f is continuous on the interval (a, b] and has an infinite discontinuity at a, then

$$\int_a^b f(x)dx = \lim_{c \to a^+} \int_c^b f(x)dx.$$

the improper integral converges if the limit exists, otherwise it diverges

3. If f is continuous on the interval [a, b], except for some c in (a, b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

the improper integral on the left diverges if either integral on the right diverges

Ex. 2: Evaluate each.

$$(a)\int_{0}^{1} \frac{4}{\sqrt{x}}\,dx$$

$$(b)\int_{-2}^{1} \frac{-2}{x^2}\,dx$$

$$(c)\int_{-2}^{0} \frac{-2}{x^2}\,dx$$

$$(d)\int_{1}^{2} \frac{4}{\sqrt{x-1}}\,dx$$

$$(e)\int_{0}^{\infty} \frac{dx}{x}$$

The. 7.5: A Special Type of Improper Integral:

$$\int_{1}^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1}, \text{ if } p > 1 \\ \text{diverges, if } p \leq 1 \end{cases}$$

Ex. 3: Find the volume and surface area of the solid formed by revolving the unbounded region lying between the graph of $f(x) = 1/x$ and the x-axis $(x \geq 1)$ about the x-axis. This solid is called Gabriel's Horn.