Plane Curves & Parametric Equations (10.2)

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Conic Sections Review (more information found in 10.1)

Circle with center (h, k) and radius r:

$$(x-h)^2+(y-k)^2=r^2$$

Ellipse with center (h, k):

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Hyperbola with center (h, k):

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}-\frac{(x-k)}{b^2}=1$$

Plane Curves & Parametric Equations

*Parametric equations introduce a third variable t, for time, into a rectangular equation that relates x and y. This allows us to refer to *when* an object is at the location of a certain point (x, y).

Definition of a Plane Curve: If f and g are continuous functions of t on an interval I, then the equations x=f(t) and y=g(t) are called p**arametric equations** and t is called the **parameter**. The set of points (x, y) obtained at t varies over the interval I is called the graph of the parametric equations. Taken together, the parametric equations and the graph are called a **plane curve**, denoted by C.

Ex. 1: Sketch the curve described by the parametric equations $x = t^2 - 4$ and $y = 2t$

$-2 \le t \le 2$.

Eliminating the Parameter

*You can eliminate the parameter t to create a rectangular equation through substitution. However, you must restrict the domain of the rectangular equation to the original boundary defined by the parameter.

Ex. 2: Sketch the curve of the parametric equations in example 1 by eliminating the parameter and adjusting the domain of the resulting rectangular equation.

*If parametric equations are given in terms of θ trigonometric functions of θ , then the parameter does not represent time but rather an angle. In this case, it is undesirable to eliminate the parameter through inverse trigonometric functions, so we will make use of trigonometric identities.

Ex. 3: Sketch the curve represented by $x = 2\sin\theta$ and $y = 5\cos\theta$, $0 \le \theta \le 2\pi$ by eliminating the parameter and finding the corresponding rectangular equation.

Finding Parametric Equations

<u>Ex. 4:</u> Find two different sets of parametric equations to represent the graph of $y = 2x^2 - 3$.