Parametric Equations & Calculus (10.3)

April 19th, 2018

Slope & Tangent Lines

Thm. 10.7: Parametric Form of the Derivative: If a smooth curve C is given by the equations x=f(t) and y=g(t), then the slope of C at (x, y) is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{dx}{dt} \neq 0$$

*You can find higher-order derivatives by applying this theorem repeatedly.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$$

Ex. 1: Find the slope and concavity (if possible) for the following parametric equations and at the given parameter.

A) $x = t^2 + 3t + 2, y = 2t; t = 0$

B) $x = \cos\theta, y = 3\sin\theta; \theta = \pi$

Arc Length

Ex. 2: Derive a formula for arc length of a curve given by the parametric equations $x=f(t)$ and $y=g(t)$.

Thm. 10.8: Arc Length in Parametric Form: If a smooth curve C is given by x=f(t) and y=g(t) such that C does not intersect itself on the interval $a \le t \le b$ (except possibly at the endpoints), then the arc length of C over the interval is given by

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{\left(f'(t)\right)^2 + \left(g'(t)\right)^2}\, dt$$

*Note that if a curve is not smooth over the entire desired interval, you may have to use several integrals to find the total arc length.

Ex. 3: Find the arc length of the curve given by the parametric equations $x = t$ and $y = \frac{t^5}{10} + \frac{1}{6t^3}$ on the interval $1 \le t \le 2$. (Be sure to graph the

curve first, to see if this can be done in one integral)