Polar Coordinates & Polar Graphs (10.4)

April 20th, 2018

Polar Coordinates

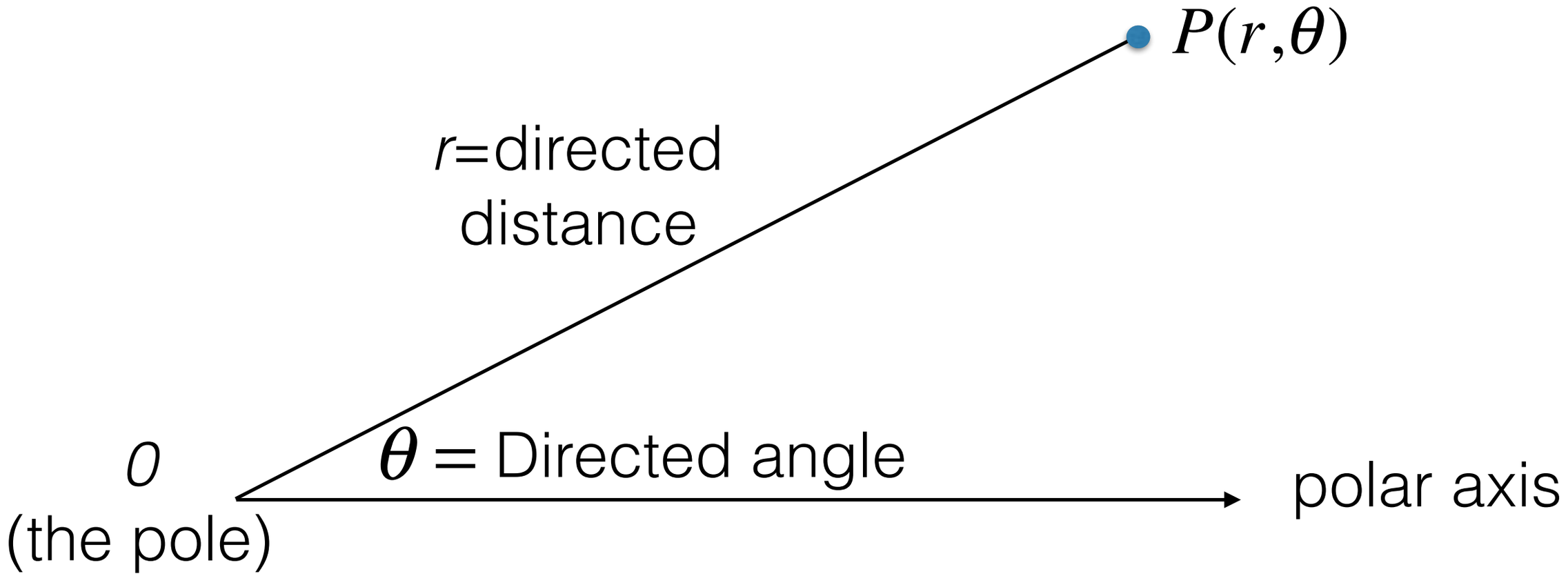
*In the **polar coordinate system**, points are of the form $P(r,\theta)$ rather than P(x, y), which is used in the rectangular coordinate system. *r* represents the directed distance from the **pole** (or origin) to point P and θ is directed angle from the **polar axis** (aligns with $\theta = 0$).

Not all polar coordinates are unique because of the cyclical nature of θ. So,

$$(r,\theta) = (r,\theta + 2\pi n)$$

and

$$(r,\theta) = (-r,\theta + \pi)$$ as -r represented the distance opposite θ


$P(r,\theta)$
r=directed distance
0
(the pole)
θ = Directed angle
polar axis

Ex. 1: Plot each of the following points on a polar coordinate plane.

A) $(2, {}^{5\pi}/_{6})$

B) $(1, {}^{-7\pi}/_{4})$

C) $(3, 1.5)$

D) $(-2, 3)$

Coordinate Conversion

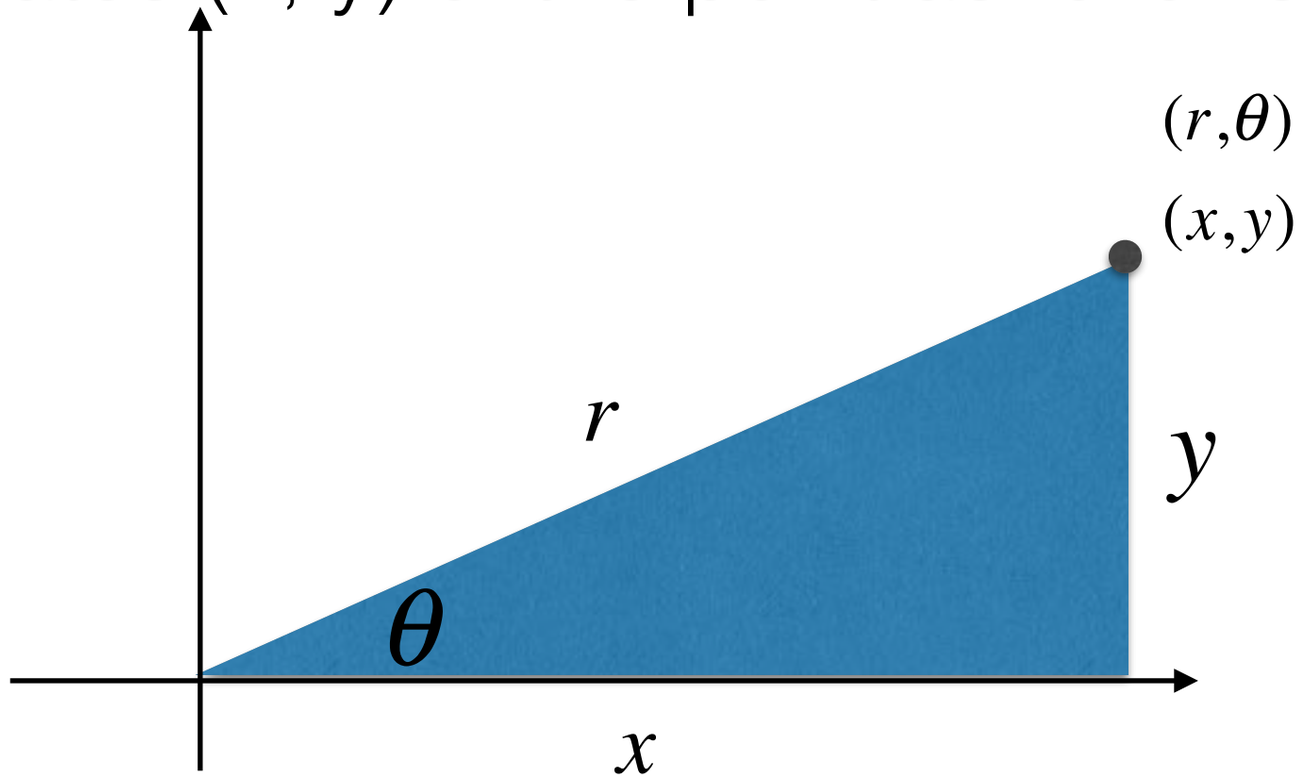
<u>Thm. 10.10: Coordinate Conversion:</u> The polar coordinates (r,θ) of a point are related to the rectangular coordinates (x, y) of the point as follows.

1. $x = r\cos\theta$

   $y = r\sin\theta$

2. $\tan\theta = \dfrac{y}{x}$

   $r^2 = x^2 + y^2$



Polar Graphs

*Polar graphs should be approached one of four ways:

1) visualize what the polar equation represents

2) convert the polar equation to rectangular form

3) use graphing calculators on polar mode

*4) make a gigantic table of values (this way is a major bummer)

Ex. 2: Graph each of the following polar equations using one of the four methods.

A) $r\sin\theta = 3$

B) $\theta = \dfrac{\pi}{4}$

C) $r = 5$

D) $r = 3\cos 2\theta$

Slope & Tangent Lines

Th. 10.11: Slope in Polar Form: If *f* is a differentiable function of θ, then the slope of the tangent line to the graph of $r = f(\theta)$ at the point (r,θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

provided that $dx/d\theta \neq 0$ at (r,θ).

Ex. 3: Find the horizontal and vertical tangent lines of

$$r = 2\sin\theta, 0 \le \theta \le \pi.$$

*Familiarize yourself with special polar graphs on page 735.