Area & Arc Length in Polar Coordinates (10.5)

April 26th, 2018

Area of a Polar Region

Thm. 10.13: Area in Polar Coordinates: If f is continuous and nonnegative on the interval $[\alpha,\beta]$, $0<\beta-\alpha\le 2\pi$, then the area of the region bounded by the graph of $r=f(\theta)$ between the radial lines $\theta=\alpha$ and $\theta=\beta$ is given by

$$A=\frac{1}{2}\int_{\alpha}^{\beta}[f(\theta)]^2\,d\theta=\frac{1}{2}\int_{\alpha}^{\beta}r^2\,d\theta$$

*This results from using the same limit definition of area on the rectangular coordinate plane that uses an infinite number of representative rectangles to set up the integral, but now using an infinite number of representative sectors.

Note that the area of a sector is given by

$$A = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2} r^2$$

Ex. 1: Find the area of the region bounded by one petal of the rose curve $r = \cos 5\theta$

Ex. 2: Find the area of the region lying between the inner and outer loops of the limacon $r = 1 - 2\cos\theta$

Points of Intersection of Polar Graphs

*You must be careful when finding points of intersection of polar equations algebraically. The only solutions you get are those that intersect at the same time, or the same value of the parameter. There may be other intersections at the pole, found by looking for points of the form

. $(0,\theta)$

Ex. 3: Find the points of intersection of the graphs of the equations below.

$$r = 2 - 3\cos\theta$$

$$r = \cos\theta$$

Area of a Region Between Two Polar Curves

Ex. 4: Use a graphing utility to graph the polar equations below and find the area of the given region.

Common interior of $r = 5 - 3\sin\theta$
and $r = 5 - 3\cos\theta$

Arc Length in Polar Form

The. 10.14: Arc Length of a Polar Curve: Let *f* be a function whose derivative is continuous on the interval $\alpha \le \theta \le \beta$. The length of the graph of. $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Ex. 5: Find the length of the curve $r = 2\alpha\cos\theta$ over the interval

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

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