Series and Convergence (9.2)

March 6th, 2018

Infinite Series

<u>Def. of Series:</u> An infinite series, or just series, of a sequence $\{a_n\}$ is

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + ... + a_k + a_{k+1} + ...$$

.

We can create a new sequence, $\{S_n\}$, where

$$S_n = a_1 + a_2 + ... + a_n = \sum_{k=1}^{n} a_k$$

and each number $S_1, S_2, ..., S_n, ...$
is a called a partial sum of the infinite series. We can determine if the series converges or diverges by determining whether the sequence of partial sums converges or diverges.

Ex. 1: Determine the convergence or divergence of the following series.

a) $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (use partial fractions to write as a telescoping series

c) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

Geometric Series

*Recall: A geometric series is a series in which each term is multiplied by a common ratio, r.

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \ldots$$

Thm. 9.6: Convergence of a Geometric Series: A geometric series with ratio r diverges if $|r| \geq 1$. If $0 < |r| < 1,$ then the series converges to the sum

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Ex. 2: Find the sum of each series.

a) $\sum_{n=1}^{\infty} 2\left(\frac{3}{4}\right)^{n-1}$

b) $1+0.1+0.01+0.001+...$

Thm. 9.7: Properties of Infinite Series:

If $\sum a_n = A, \sum b_n = B,$ and c is a real number, then the following series converge to the indicated sums.

1. $$\sum_{n=1}^{\infty} ca_n = cA$$

2. $$\sum_{n=1}^{\infty} \left(a_n + b_n\right) = A + B$$

3. $$\sum_{n=1}^{\infty} \left(a_n - b_n\right) = A - B$$

nth-Term Test for Convergence

Thm. 9.8: Limit of nth Test of a Convergent Series:

If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

Thm. 9.9: nth-Term Test for Divergence:

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Ex. 3: Determine the convergence or divergence of each series.

a) $\sum_{n=1}^{\infty}(-5n+3)$

b) $\sum_{n=1}^{\infty}\frac{n!}{(n+1)!}$

c) $\sum_{n=1}^{\infty}e^{-n}$