The Integral Test & p-Series (9.3)

March 8th, 2018

The Integral Test

Thm. 9.10: The Integral Test: If f is positive, continuous, and decreasing for $x \geq 1$ and. $a_n = f(n)$, then

$$\sum_{n=1}^{\infty} a_n \qquad \text{and} \qquad \int_{1}^{\infty} f(x)\,dx$$

either both converge or diverge.

Ex. 1: Use the Integral Test to determine the convergence or divergence of the series.

a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (Harmonic Series)

b) $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

p-Series & Harmonic Series

Thm. 9.11: Convergence of p-Series: The p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$$

1. converges if p>1, and

2. diverges if $0 < p \leq 1$.

Ex. 2: Use the Integral Test to determine the convergence or divergence of the p-series.

a) $\sum_{n=1}^{\infty} \frac{5}{n^4}$

b) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$