

Comparison of Series

(9.4)

March 9th, 2018

Direct Comparison Test

Thm. 9.12: Direct Comparison Test:

Let $0 < a_n \leq b_n$ for all n .

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Ex. 1: Determine the convergence or divergence of each series by applying the direct comparison test.

a)
$$\sum_{n=1}^{\infty} \frac{n}{5n^2 - 4}$$

b)
$$\sum_{n=1}^{\infty} \frac{n}{2^n (n+1)}$$

c)
$$\sum_{n=3}^{\infty} \frac{1}{(n-2)^2}$$

Limit Comparison Test

Thm. 9.13: Limit Comparison Test: Suppose that $a_n > 0$,

$$b_n > 0, \text{ and } \lim_{n \rightarrow \infty} \left(\frac{a_n}{b_n} \right) = L$$

where L is finite and positive. Then the two series $\sum a_n$
and $\sum b_n$

either both converge or both diverge.

Ex. 2: Determine the divergence or convergence of each series by applying the limit comparison test.

a)
$$\sum_{n=3}^{\infty} \frac{1}{(n-2)^2}$$

b)
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

c)
$$\sum_{n=1}^{\infty} \frac{1}{2n+7}$$