Alternating Series (9.5)

March 12th, 2018

Alternating Series

Thm. 9.14: Alternating Series Test: Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty}(-1)^n a_n \qquad \text{and} \qquad \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

converge if the following two conditions are met.

1. $\lim_{n\to\infty} a_n = 0$

2. $a_{n+1} \le a_n$, for all n

Ex. 1: Determine the divergence or convergence of each of the following series.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n}$

b) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n!} \right)$

c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{2^n}$

Alternating Series Remainder

Thm. 9.15: Alternating Series Remainder: If a convergent alternating series satisfies the condition $a_{n+1} \le a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is less than (or equal to) the first neglected term. That is,

$$\left|S - S_N\right| = \left|R_N\right| \le a_{N+1}.$$

Ex. 2: Approximate the sum of the following series by using the first six terms.

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n!}\right)$$

Ex. 3: Consider the convergent alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$

How many terms should be taken so that the error does not exceed 0.0005?

Absolute and Conditional Convergence

Thm. 9.16: Absolute Convergence: If the series $\sum|a_n|$ converges, then the series $\sum a_n$ also converges.

Definitions of Absolute and Conditional Convergence:

1. $\sum a_n$ is absolutely convergent if $\sum|a_n|$ converges.
2. $\sum a_n$ is conditionally convergent if $\sum a_n$ converges but $\sum|a_n|$ diverges.

Ex. 4: Determine whether each series converges conditionally or absolutely, or diverges.

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

b) $\sum_{n=1}^{\infty} (-1)^{n+1} \arctan n$