The Ratio Test (9.6)

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The Ratio Test

<u>The. 9.17: The Ratio Test:</u> Let $\sum a_n$ be a series with nonzero terms.

1. $\sum a_n$ converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

2. $\sum a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$

3. The Ratio Test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

*Guidelines for Testing a Series for Convergence or Divergence

1. Does the nth term approach 0? If not, the series diverges.
2. Is the series one of the special types-geometric, p-series, telescoping, or alternating?
3. Can the Integral Test or Ratio Test be applied?
4. Can the Direct Comparison Test or Limit Comparison Test be used with one of the favorable types of series (like those from #2 above)?

Ex. 1: Determine the convergence/divergence of each of the following series using the Ratio Test, if applicable. If inconclusive, use a different test learned previously.

(a) $\sum \frac{n^n}{n!}$

(b) $\sum \frac{1}{n^2}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$

(d) $\sum \frac{2^n}{n!}$

***Put a page marker on page 644 where you will find a current summary of tests for series convergence and divergence. Refer to it regularly but do not use the Root Test as it is not a standard of Calculus BC.