# Taylor Polynomials & Approximation (9.7)

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# Taylor & Maclaurin Polynomials

Definitions of nth Taylor Polynomial and nth Maclaurin Polynomial:

If *f* has *n* derivatives at *c*, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the **nth Taylor Polynomial for *f* at *c*.**

If c=0, then

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

is also called the **nth Maclaurin Polynomial for f.**

*The Taylor Polynomial can be used to compare an elementary function f(x) to a infinite polynomial expansion P(n), or series, by using two functions that have that pass through the same point (c,f(c)) and have the same slope (c, f(c)).

*Ex. 1:* Find the nth-Maclaurin Polynomial for $e^x$ .

*Ex. 2: Find the nth Maclaurin Polynomial for $\dfrac{1}{1-x}$ .

*Ex. 3:* Find the nth Maclaurin Polynomial for $\sin x$ .

*Ex. 4:* Find the nth Maclaurin Polynomial for $\cos x$ .

Ex. 5: Find the third Taylor polynomial for $f(x) = \cos x$, expanded about $x = \pi / 3$ .

Graph each to compare.

# Remainder of a Taylor Polynomial

*The Taylor polynomial allows us to approximate a particular value of f(x) within a certain amount of error given by the remainder, $R_n(c)$ .

$$f(x) = P_n(x) + R_n(x)$$

Exact value=Approximate value + Remainder

$$error = \left|R_n(x)\right| = \left|f(x) - P_n(x)\right|$$

*Thm. 9.19: Taylor's Theorem:* If a function $f$ is differentiable through order $n+1$ in an interval $I$ containing $c$, then, for each $x$ in $I$, there exists $z$ between $x$ and $c$ such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x)$$

where

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$$

Ex. 6: Use Taylor's Theorem to obtain an upper bound for the error of the approximation given by

$$e \approx 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!}$$

Then calculate the exact value of the error.

Ex. 7: Determine the degree of the Maclaurin polynomial required for the error in approximation of the function at the indicated value of x to be less than 0.001.

$$e^{0.3}$$