Representation of Functions by Power Series (9.9)

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Geometric Power Series

*The Geometric Series $\sum_{n=0}^{\infty} ar^n = \dfrac{a}{1-r}, |r| < 1$

can be represented by the function $f(x) = \dfrac{1}{1-x} = \sum_{n=0}^{\infty} ar^n$

on the interval of convergence (-1, 1)

because f(x) so closely resembles the sum $\dfrac{a}{1-r}$.

To find a power series for a similar function (with its own interval of convergence, just rewrite it in the form $\dfrac{a}{1-r}$
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Operations with Power Series

Let $f(x)=\sum a_n x^n$ and $g(x)=\sum b_n x^n$.

1. $f(kx)=\sum_{n=0}^{\infty} a_n k^n x^n$

2. $f(x^N)=\sum_{n=0}^{\infty} a_n x^{nN}$

3. $f(x)\pm g(x)=\sum_{n=0}^{\infty} (a_n \pm b_n)x^n$

Ex. 1: Find a power series for each function, centered at c, and determine the interval of convergence.

a) $f(x) = \frac{3}{2x-1}, c = 0$

b) $f(x) = \frac{3}{2x-1}, c = 2$

c) $f(x) = \frac{4x-7}{2x^2+3x-2}, c = 0$

Finding a power Series by Integration

Ex. 2: Find a power series for $g(x) = \arctan x$, centered at 0.