Motion Along a Line: Vectors

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Parametric Equations- Notation

If a smooth curve C is given by x=f(t) and y=g(t), then

$\frac{dy}{dx}$ represents the slope of C at the point (x, y), or the slope of the path of a particle traveling along C, or the rate of change of C with respect to x.

$x'(t)=\frac{dx}{dt}$ is the rate at which the x-coordinate is changing with respect to t (velocity of the particle in the horizontal direction).

$y'(t)=\frac{dy}{dt}$ is the rate at which the y-coordinate is changing with respect to t (velocity of the particle in the vertical direction).

Vectors & Calculus

$\langle x(t), y(t) \rangle$ is the position vector at any time t.

$\langle x'(t), y'(t) \rangle$ is the velocity vector at any time t.

$\langle x''(t), y''(t) \rangle$ is the acceleration vector at any time t.

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ is the speed of the particle or magnitude (length) of the velocity vector.

$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ is the arc length of the curve C from t=a to t=b, or the distance traveled by the particle from time [a,b].

Ex. 1: A particle moves in the xy-plane so that at any time t, the position of the particle is given by

$$x(t) = t^3 + 4t^2; y(t) = t^4 - t^3.$$

(a) Find the velocity vector when t=1.

(b) Find the acceleration vector when t=2.

Ex. 2: A particle moves in the xy-plane so that at any time t, $t \geq 0$, the position of the particle is given by

$x(t) = t^2 + 3t, y(t) = t^3 - 3t^2$. Find the magnitude of the velocity vector when t=1.

Ex. 3: A particle moves in the xy-plane so that $x = \sqrt{3} - 4\cos t$ and $y = 1 - 2\sin t$, where $0 \le t \le 2\pi$. The path of the particle intersects the x-axis twice. Find the distance traveled by the particle between the two x-intercepts. (You will need to use a calculator).

Ex. 4: An object moving along a curve in the xy-plane has position ((x(t), y(t)) at time t with $\frac{dx}{dt} = \sin\left(t^3\right), \frac{dy}{dt} = \cos\left(t^2\right)$. At time t=2, the object is at the position (1, 4).

(a) Find the acceleration vector for the particle at t=2.
(b) Write the equation of the tangent line to the curve at the point where t=2.
(c) Find the speed of the vector at t=2.
(d) Find the position of the particle at time t=1.

*The examples and definitions in this presentation are courtesy of Nancy Stephenson of Clements High School in Sugar Land, Texas, from the “Vectors in AP Calculus”.