

The Natural Logarithmic Function: Integration (4.6)

February 28th, 2019

I. Log Rule for Integration

Thm. 4.19: Log Rule for Integration: Let u be a differentiable function of x .

$$1. \quad \int \frac{1}{x} dx = \ln|x| + C$$

$$2. \quad \int \frac{1}{u} du = \ln|u| + C$$

$$\Rightarrow \int \frac{u'}{u} dx = \ln|u| + C$$

Ex. 1: Find each indefinite integral.

a. $\int \frac{3}{x+6} dx$

b. $\int \frac{6x}{x^2+2} dx$

c. $\int \frac{e^{-x}}{e^{-x}+1} dx$

d. $\int \frac{2x-1}{2x^2-2x} dx$

Dividing Before Integrating

Ex. 2: Find each indefinite integral.

a) $\int \frac{2x^2 - 3x + 1}{x - 1} dx$

b) $\int \frac{4x^3 - 2x + 5}{x^2 + 2} dx$

c) $\int \frac{3x^2 - 3x + 5}{x^2 - 2} dx$

Ex. 3: Find the area of the region bounded by the graphs of the equations $y = \frac{x^2 - 3x + 5}{3x^2}$, $x=1$, $x=4$, and $y=0$.

*Guidelines for Integration

1. Know the 20 basic integration formulas you've already learned: the power rule, the log rule, and the 10 trigonometric rules.
2. Try to recognize which of those formulas best matches the integrand, and choose u accordingly.
3. If nothing fits, try to manipulate the integrand using algebra or trigonometric identities.

Ex. 4: Solve the differential equation $\frac{dy}{dx} = \frac{3}{x \ln x}$.

II. Integrals of the Trigonometric Functions

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

Ex. 5: Find $\int \cot 5\theta d\theta$.

Ex. 6: Find the average value of $f(x) = \frac{\tan x}{\sin x}$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.