Extrema on an Interval (3.1)

December 6th, 2018

I. extrema of a function

Def. of Extrema: Let f be defined on an interval I containing c.

1. f(c) is the minimum of f on I if $f(c) \leq f(x)$ for all x in I.

2. f(c) is the maximum of f on I if $f(c) \geq f(x)$ for all x in I.

The maximum & minimum of a function on an interval are the extreme values, or extrema of the function on the interval.

Thm. 3.1: The Extreme Value Theorem: If f is continuous on a closed interval [a, b], then f has both a minimum and a maximum on the interval.

II. Relative Extrema & Critical Numbers

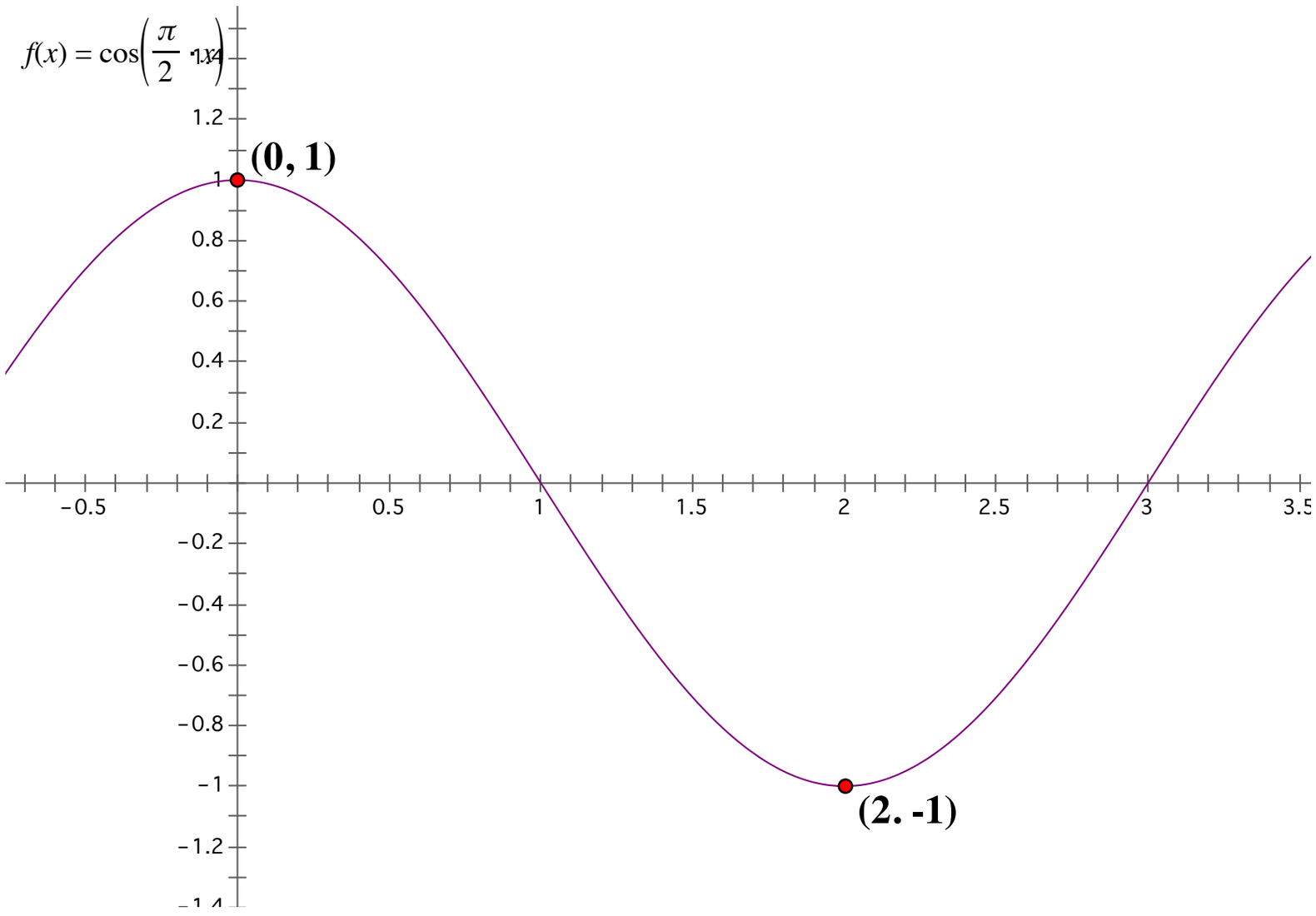
*All high points of a function are called relative maxima & all low points are called relative minima. The highest of all high points is called the absolute maximum & the lowest of all low points is called the absolute minimum. When any extrema occur at a point where the graph is curved, the graph has a horizontal tangent line at that point. When any extrema occur at a point where the graph is a sharp peak, the function is not differentiable at that point.

Def. of Relative Extrema:

1. If there exists an open interval containing c on which f(c) is a maximum, then f(c) is called a relative maximum of f, or f has a relative maximum at (c, f(c)).

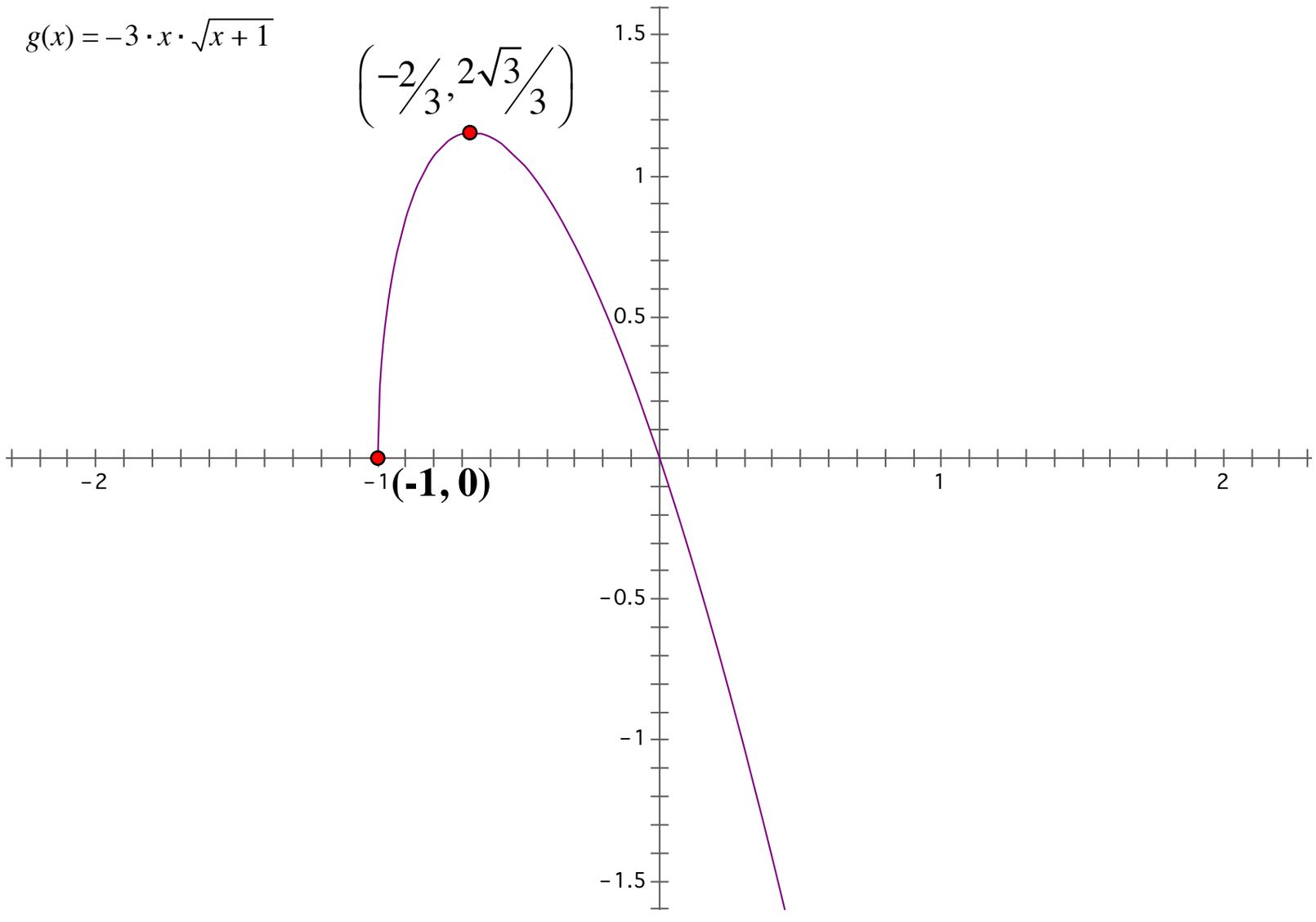
2. If there exists an open interval containing c on which f(c) is a minimum, then f(c) is called a relative minimum of f, or f has a relative minimum at (c, f(c)).

Ex. 1: Find the value of the derivative (if it exists) of $f(x)=\cos\frac{\pi x}{2}$ at each indicated extremum.

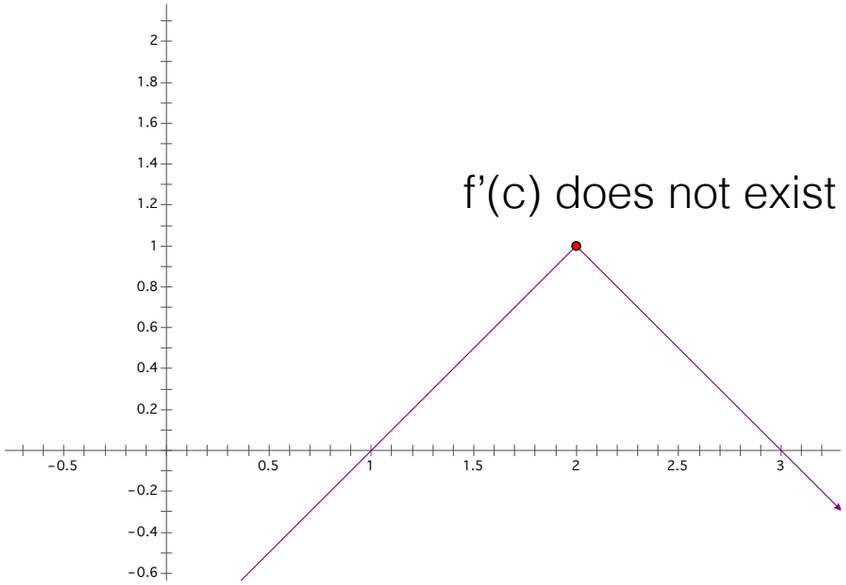


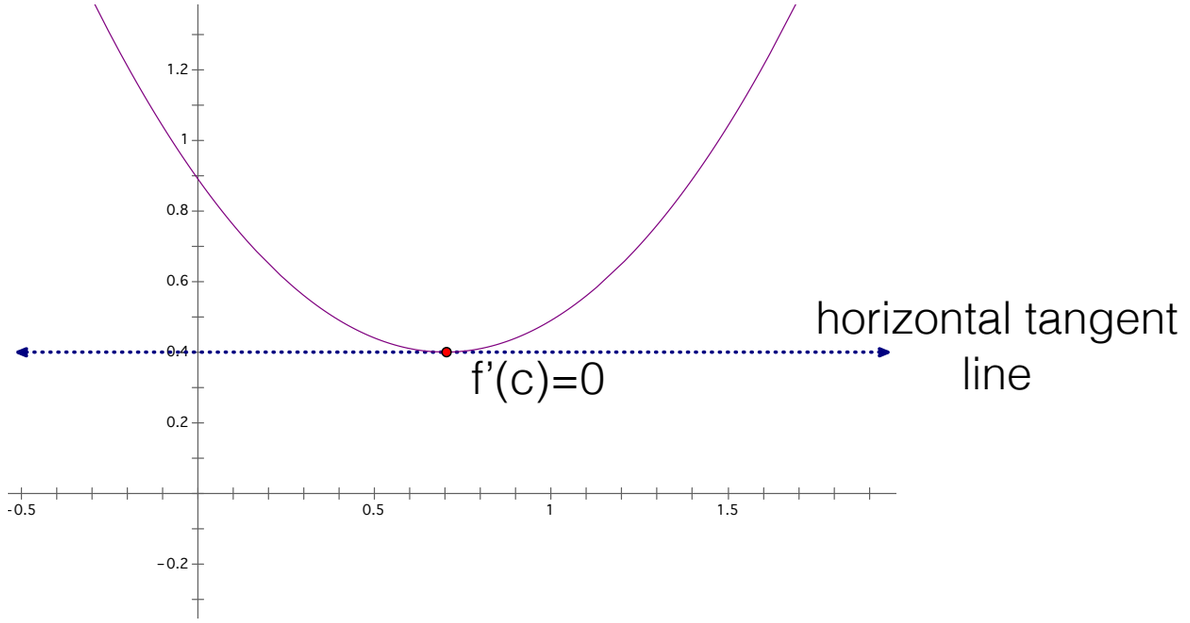
<u>You Try:</u> Find the value of the derivative (if it exists) of at each indicated extremum.

$$f(x) = -3x\sqrt{x+1}$$



Def. of a Critical Number: Let f be defined at c. If f'(c)=0 or if f is not differentiable at c, then c is a critical number of f.





Thm. 3.2: Relative Extrema Occur Only at Critical Numbers: If f has a relative minimum or relative maximum at x = c, then c is a critical number of f.

Ex. 2: Find any critical numbers of the function $g(x) = x^2\left(x^2 - 4\right)$

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<u>You Try:</u> Find any critical numbers of the function $f(x)=\frac{4x}{x^2+1}$

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III. Finding Extrema on a Closed Interval

<u>Guidelines for Finding Extrema on a Closed Interval:</u> To find the extrema of a continuous function f on a closed interval [a, b]:

1. Find the critical numbers of f on (a, b).

2. Evaluate f at each critical number in (a, b).

3. Evaluate f(a) and f(b), the endpoints.

4. The least of these values is the minimum. The greatest is the maximum.

Ex. 3: Locate the absolute extrema of the function $g(x)=\sqrt[3]{x}$ on the interval [-1, 1].

You Try: Locate the absolute extrema of the function $f(x) = \sin 3x$ on the interval $\left[-\pi/6, \pi/3\right]$.

Ex. 4: Find the absolute extrema of $f(x) = x^2 e^x$ on $(-3,2)$