Rolle's Theorem & the Mean Value Theorem (3.2)

December 10th, 2018

I. Rolle's Theorem

Thm. 3.3: Rolle's Theorem: Let f be continuous on the closed interval [a, b] and differentiable on the open interval (a, b). If f(a) = f(b), then there is at least one number c in (a, b) such that f'(c)=0.

Ex. 1: Determine whether Rolle's Theorem can be applied to $f(x)=x^2-5x+4$ on the closed interval [1, 4]. If Rolle's Theorem can be applied, find all values c in the open interval (1, 4) such that f'(c)=0.

II. The Mean Value Theorem

Thm. 3.4: The Mean Value Theorem: If f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Ex. 2: Determine whether the Mean Value Theorem can be applied to $f(x) = x(x^2 - x - 2)$ on the closed interval [-1, 1]. If the Mean Value Theorem can be applied, find all the values of c in the open interval (-1, 1) such that $f'(c) = \frac{f(1) - f(-1)}{1 - (-1)}$.

<u>You Try:</u> Determine whether the Mean Value Theorem can be applied to $f(x)=x^3$ on the closed interval [0, 1]. If the Mean Value Theorem can be applied, find all values of c in the open interval (0, 1) such that

$$f'(c)=\frac{f(1)-f(0)}{1-0}$$.

Ex. 3: At 9:13am, a sports car is traveling 35 miles per hour. Two minutes later, the car is traveling 85 miles per hour. Prove that at some time during this two-minute interval, the car's acceleration is exactly 1500 miles per hour squared.