Increasing & Decreasing Functions & The First Derivative Test (3.3)

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I. Increasing & Decreasing Functions

Defs. of Increasing & Decreasing Functions: A function f is increasing on an interval if for any two numbers x_1 and x_2 in the interval $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

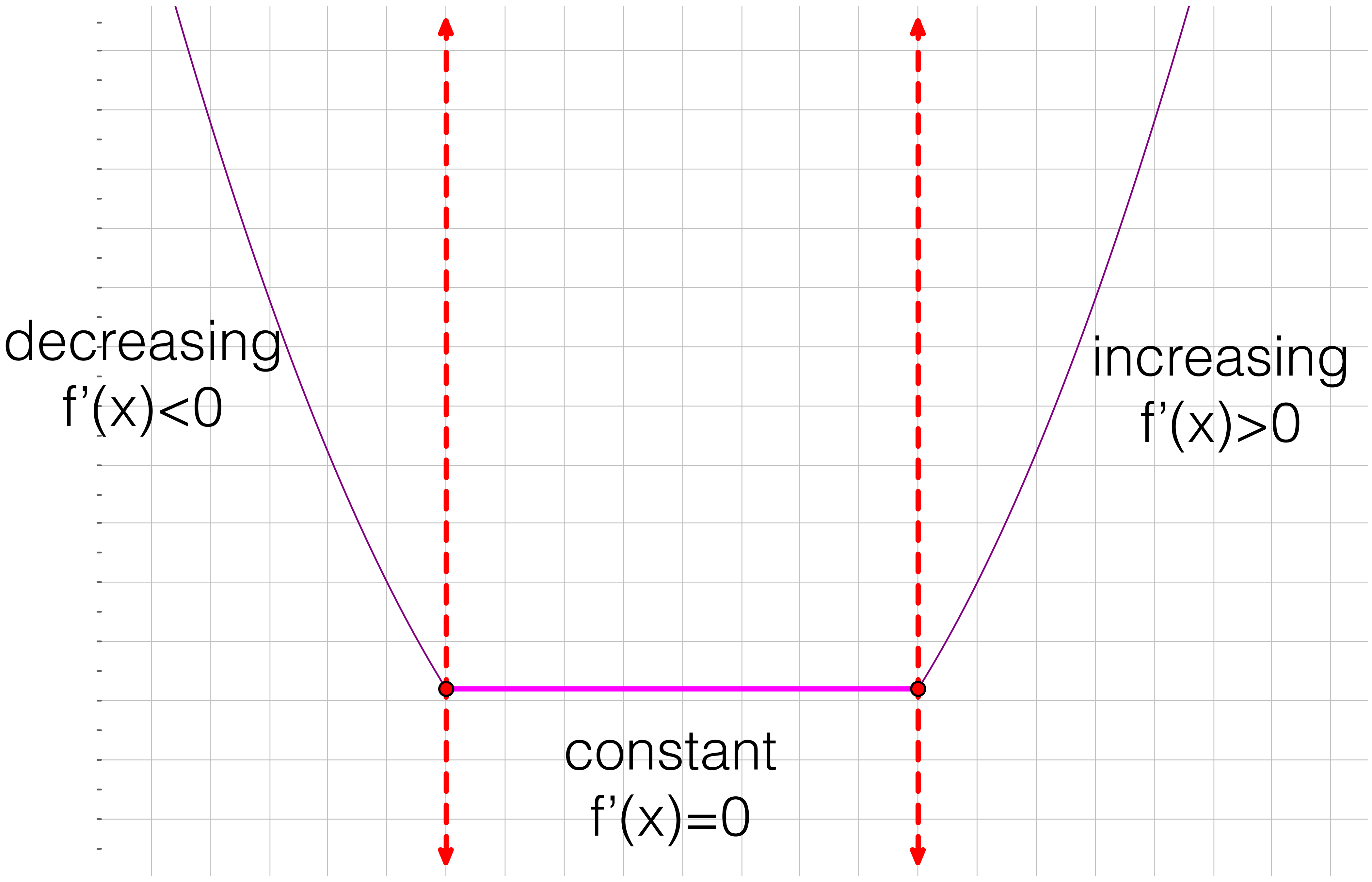
A function f is decreasing on an interval if for any two numbers x_1 and x_2 in the interval $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

*Increasing is going up from left to right, decreasing is going down from left to right.

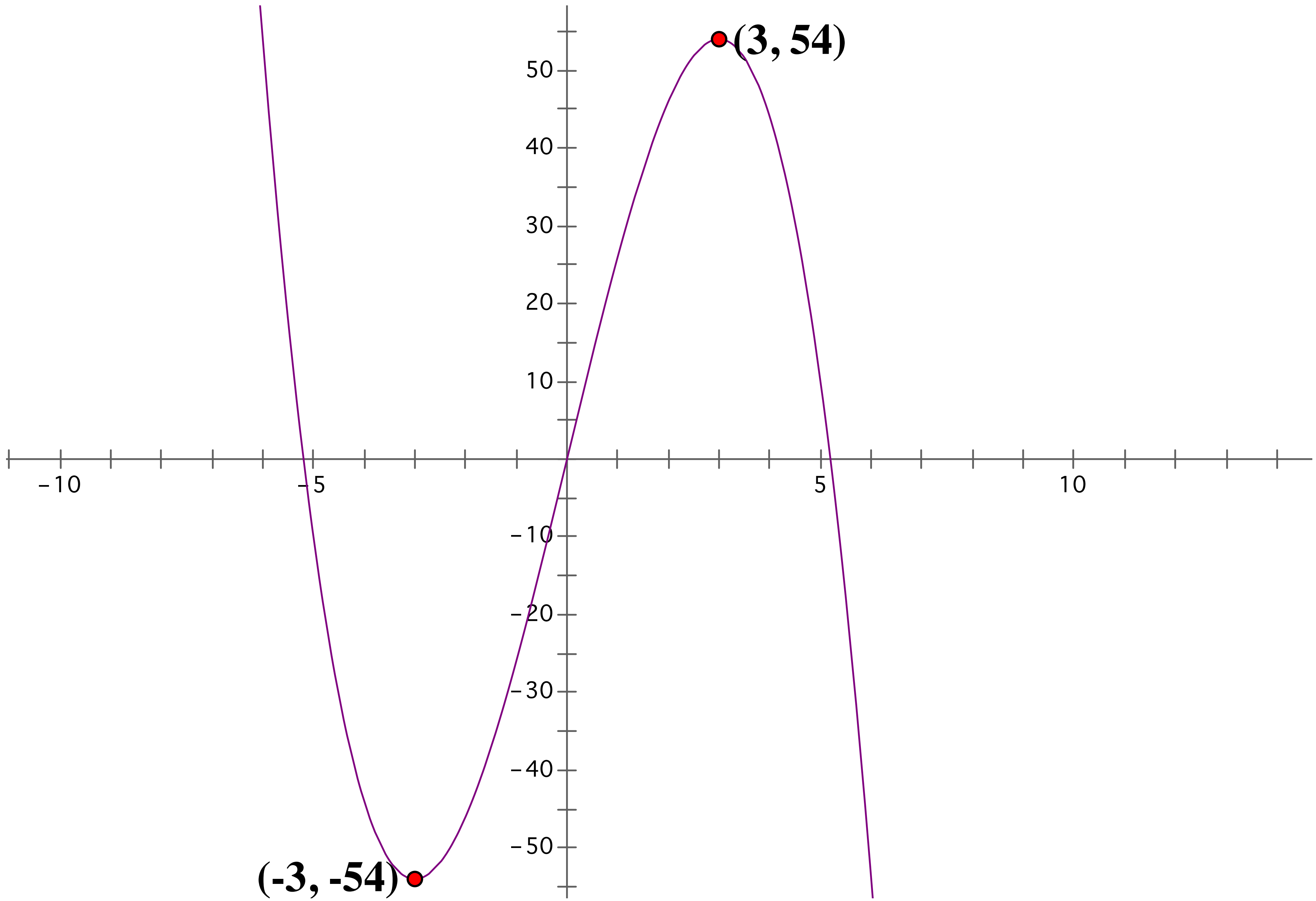
Thm. 3.5: Test for Increasing & Decreasing Functions:

Let f be a function that is continuous on the closed interval [a, b] and differentiable on the open interval (a, b).

1. If $f'(x)>0$ for all x in (a, b), then f is increasing on [a, b], (because f has a positive slope on (a, b)).
2. If $f'(x)<0$ for all x in (a, b), then f is decreasing on [a, b], (because f has a negative slope on (a, b)).
3. If $f'(x)=0$ for all x in (a, b), then f is constant on [a, b], (because f is a horizontal line & has a slope of 0 on (a, b)).


decreasing
f'(x)<0
increasing
f'(x)>0
constant
f'(x)=0

Ex. 1: Find the open intervals on which $h(x) = 27x - x^3$ is increasing or decreasing.



(3, 54)
(-3, -54)
50
40
30
20
10
−10
−20
−30
−40
−50
−10
−5
5
10

Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing: Let f be continuous on the interval (a, b). To find the open intervals on which f is increasing or decreasing, use the following steps.

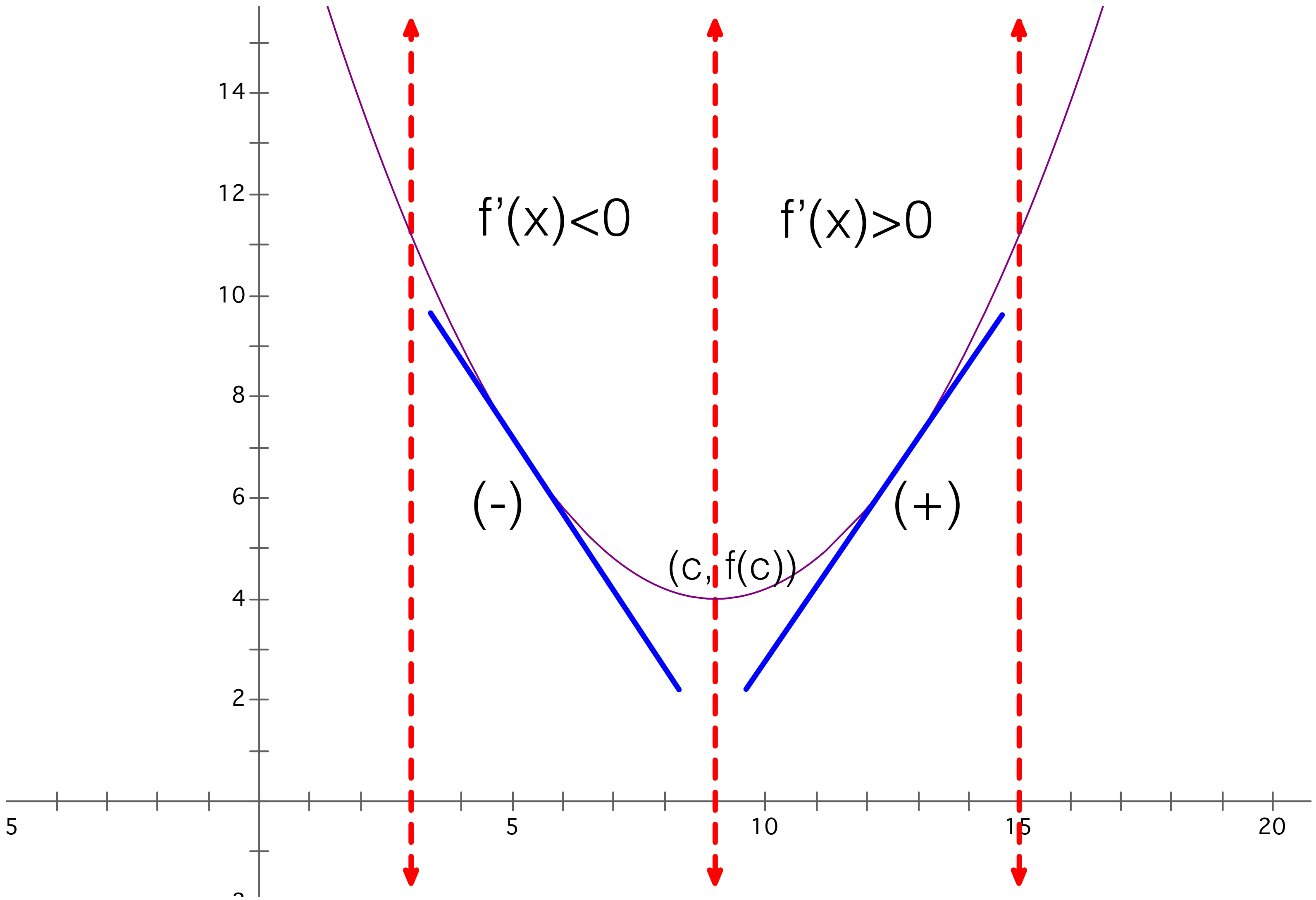
1. Locate the critical numbers of f in (a, b) and use these numbers to determine the test intervals.
2. Determine the sign of f'(x) at one test value in each interval.
3. Use Thm. 3.5 to determine whether f is increasing or decreasing on each interval.

You Try: Identify the open intervals on which the function $y = x^2 - \ln x$ is increasing or decreasing.

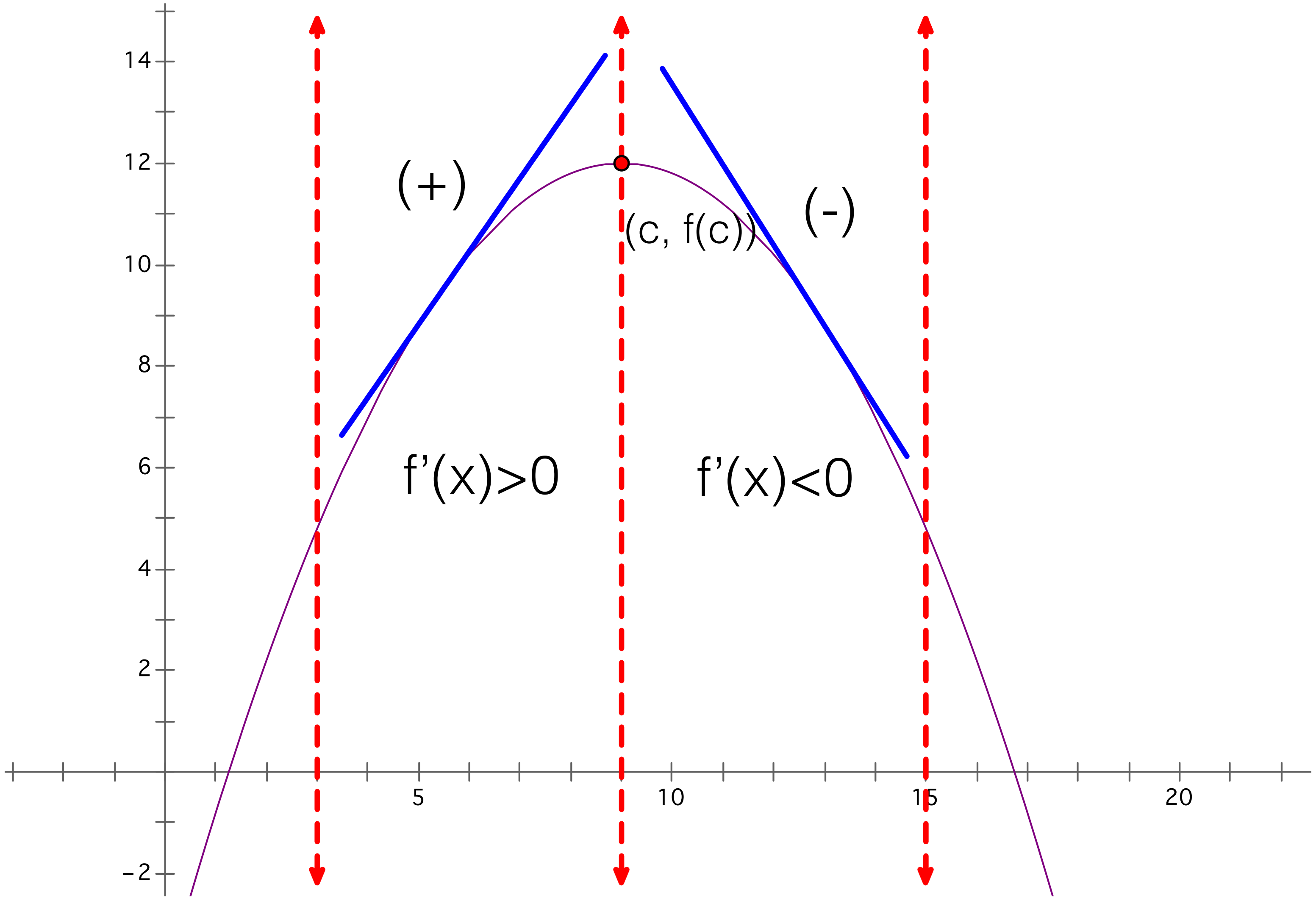
II. The First Derivative Test

Thm. 3.6: The First Derivative Test: Let c be a critical number of a function f that is continuous on the open interval I containing c. If f is differentiable on the interval, except possibly at c, then f(c) can be classified as follows.

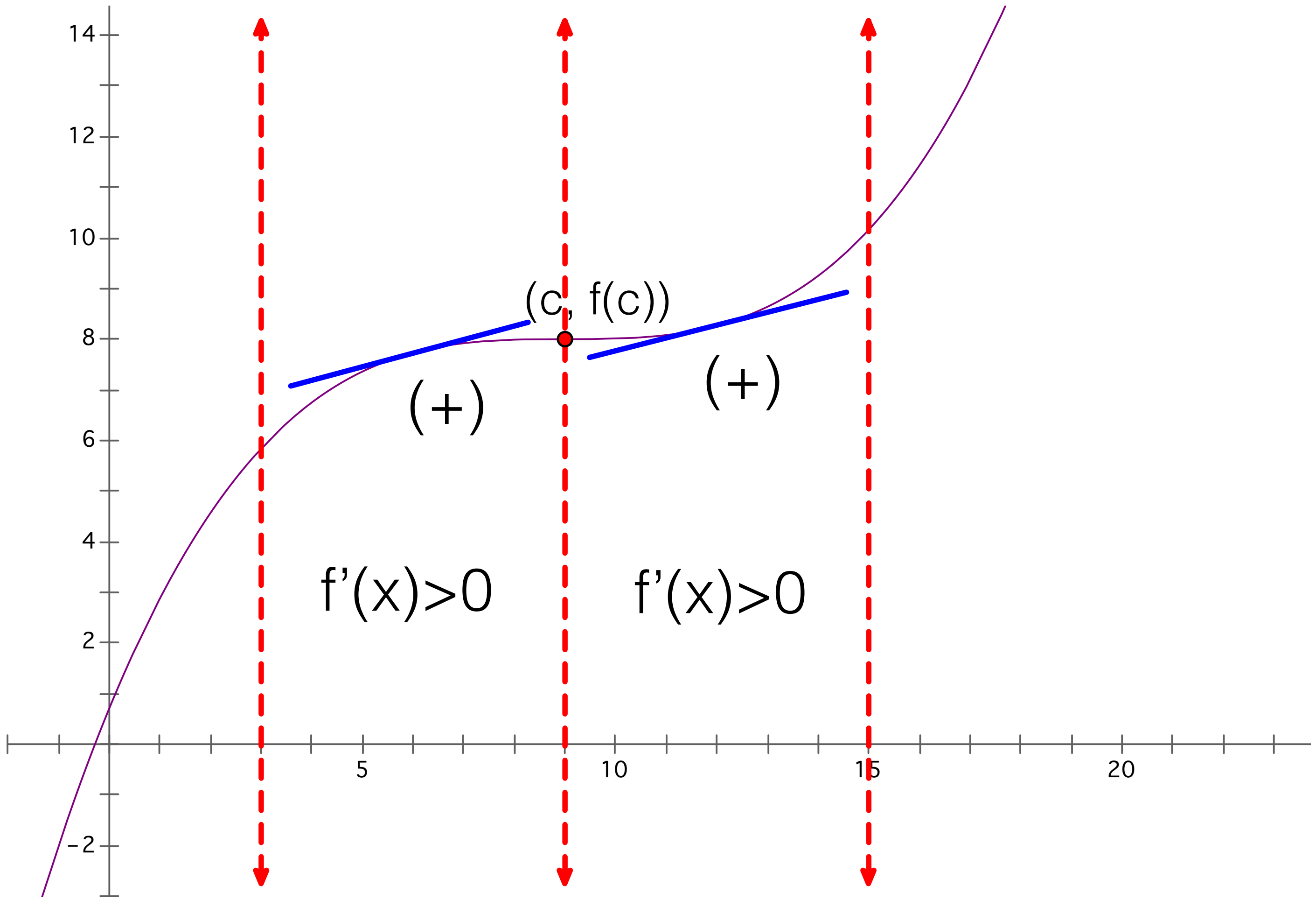
1. If f’(x) changes from negative to positive at c, then f has a relative minimum at (c, f(c)).



2. If f’(x) changes from positive to negative at c, then f has a relative maximum at (c, f(c)).



3. If f'(x) is positive on both sides of c or negative on both sides of c, then f(c) is neither a relative minimum nor a relative maximum.



Ex. 2: Find the relative extrema of the function $f(x) = \sin x \cos x$ on the interval $[0, 2\pi)$.

You Try: Find the relative extrema of the function

$$f(x) = \frac{x}{x+1}$$