# Concavity & The Second Derivative Test (3.4)

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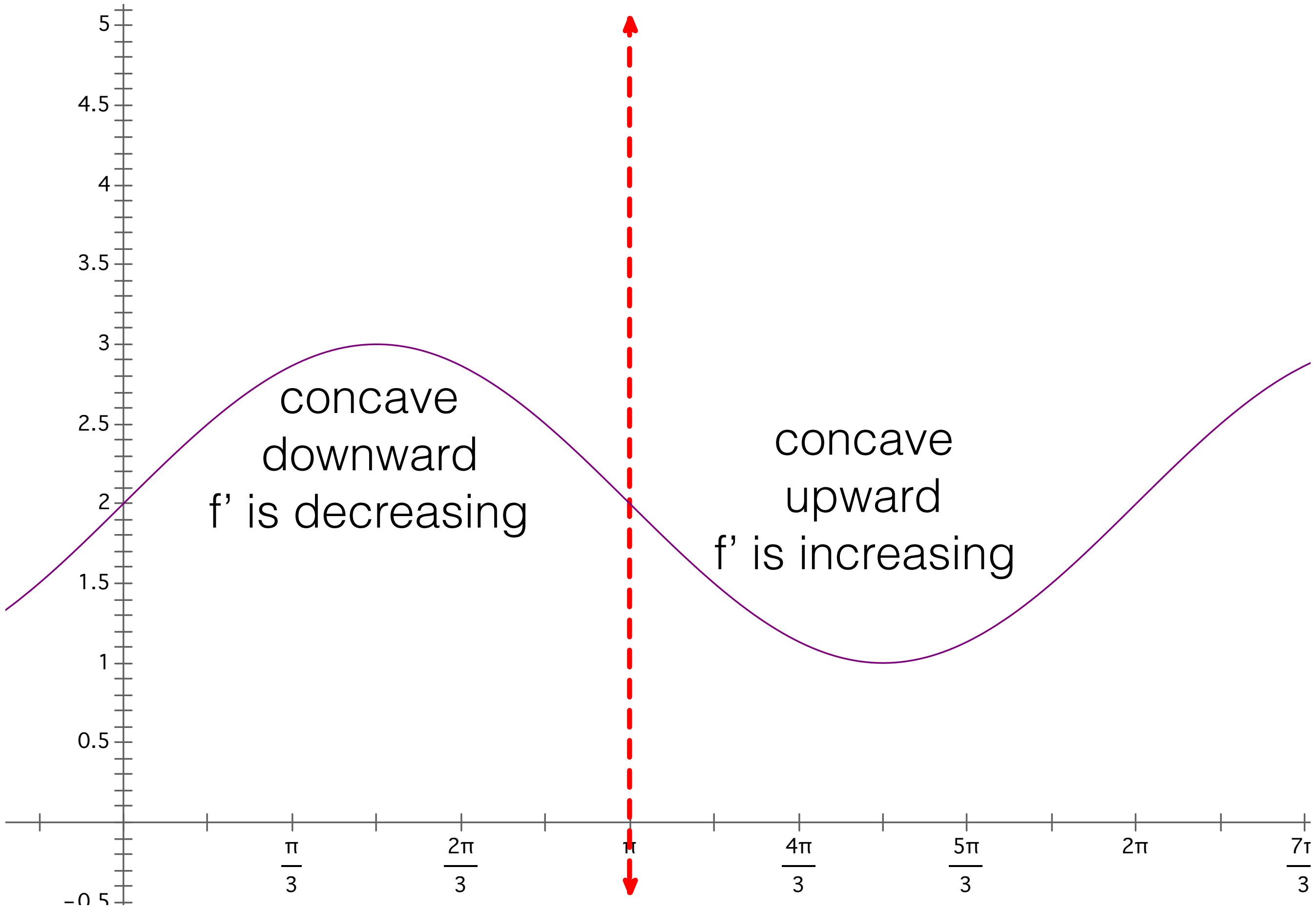
# I. concavity

Def. of Concavity: Let f be differentiable on an open interval I. The graph of f is concave upward on I if f' (the slope) is increasing on the interval and concave downward on I if f' (the slope) is decreasing on the interval.



Thm. 3.7: Test for Concavity: Let f be a function whose second derivative exists in the open interval I.

1. If $f''(x)>0$ for all x in I, then the graph of f is concave upward on I.

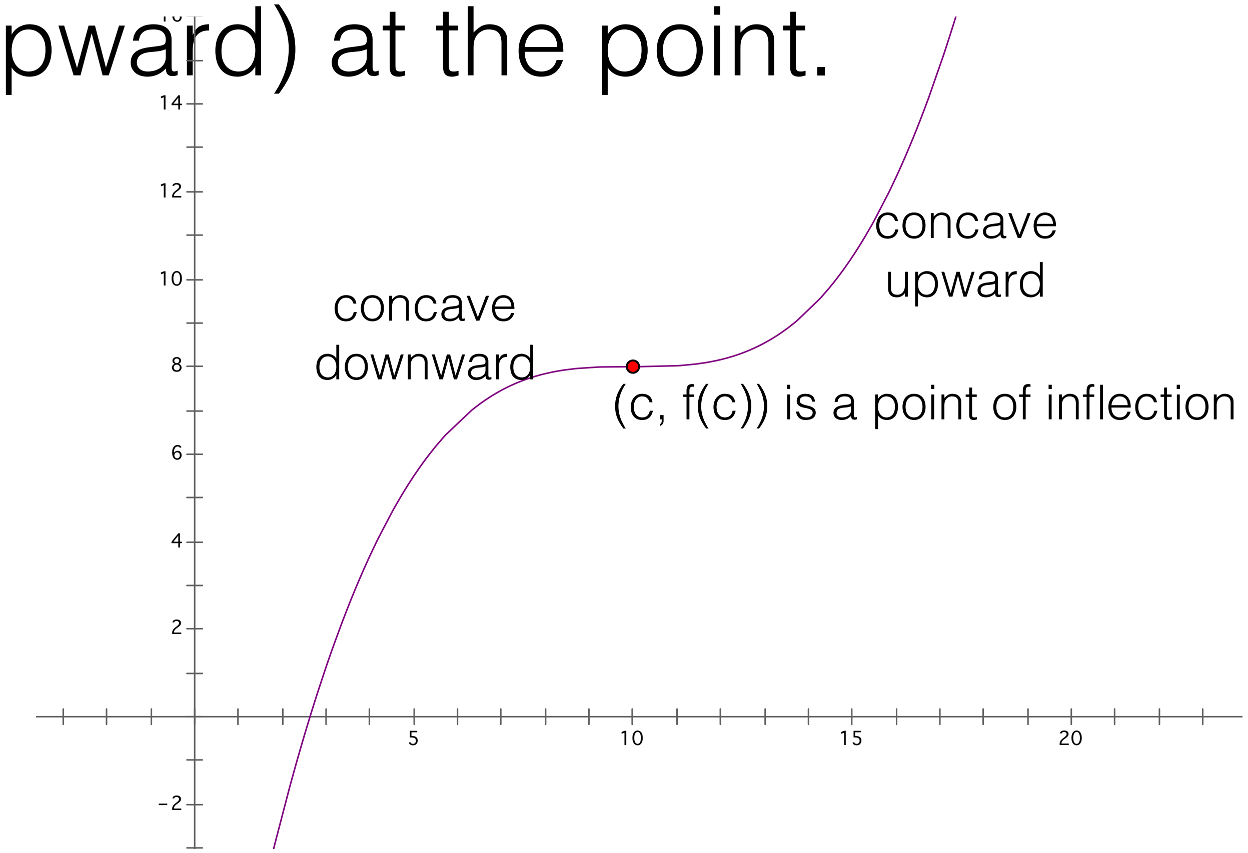
2. If $f''(x)<0$ for all x in I, then the graph of f is concave downward on I.

Ex. 1: Determine the open intervals on which the graph of $y = x^5 - 5x + 2$ is concave upward or concave downward.

<u>You Try:</u> Determine the open intervals on which the graph of $f(x) = \sin x - \cos x$ is concave upward or concave downward on the interval $[0, 2\pi]$ .

# II. Points of inflection

<u>Def. of Points of Inflection:</u> Let f be a function that is continuous on an open interval and let c be a point in the interval. If the graph of f has a tangent line at the point (c, f(c)), then this point is a <u>point of inflection</u> of the graph of f if the concavity of f changes from upward to downward (or downward to upward) at the point.



Thm. 3.8: Points of Inflection: If (c, f(c)) is a point of inflection of the graph of f, then either $f''(c)=0$ or $f''(c)$ does not exist.

Ex. 2: Find the points of inflection and discuss the concavity of the graph of the function $f(x) = \ln(3x^2 + 2)^2$

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You Try: Find the points of inflection and discuss the concavity of the graph of the function $f(x) = 2x^3 - 3x^2 - 12x + 5$

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# III. The second derivative test

Thm. 3.9: Second Derivative Test: Let f be a function such that $f'(c)=0$ and the second derivative of f exists on the open interval containing c.

1. If $f''(c)>0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c)<0$, then f has a relative maximum at $(c, f(c))$.

If $f''(c)=0$, the test fails. When this occurs, you do not have enough information to determine whether f(c) is a minimum or maximum value. Thus, you must use the First Derivative Test.

<u>Ex. 3:</u> Find the relative extrema of $f(x) = -(x-5)^2$ . Use the Second Derivative Test where applicable.

You Try: Find the relative extrema of $f(x) = \frac{-3x^2}{2x+1}$. Use the Second Derivative Test where applicable.