Differentials (3.7)

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I. Differentials

*The process of using the derivative of a function y with respect to x, or $\frac{dy}{dx}$, to find the equation of the tangent line at a given point on the function is to find a linear approximation of the function at that point. dy $(\approx \Delta y)$ is the <u>differential of y</u> and dx= Δx is the <u>differential of x.</u>

Def. of Differentials: Let y=f(x) be a function that is differentiable on an open interval containing x. The differential of x, or dx, is any nonzero real number. The differential of y, or dy, is $dy = f'(x)dx$

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*This results from solving the identity

$$\frac{dy}{dx} = f'(x)$$

*dy is an approximation of Δy, where

$$\Delta y = f(c + \Delta x) - f(c)$$

for a point (c, f(c)) on the function.

Ex. 1: Given the function $y = 1 - 2x^2$, evaluate and compare Δy and dy when x = 0 and Δx = dx= -0.1.

You Try: Given the function $y=2x+1$, evaluate and compare Δy and dy when $x = 2$ and $\Delta x = dx = 0.01$.

II. Calculating Differentials

*If u and v are both differentiable functions of x, then their differentials are du=u’dx and dv=v’dx. Thus, all the differentiation rules can be written in <u>differential form.</u>

Differential Formulas:

1. Constant Multiple: $d[cu] = cdu$

2. Sum or Difference: $d[u \pm v] = du \pm dv$

3. Product: $d[uv] = udv + vdu$

4. Quotient: $d\left[\frac{u}{v}\right] = \frac{vdu - udv}{v^2}$

Ex. 2: Find the differential dy of each function.

a. $y = 3x^{2/3}$

b. $y = \sqrt{9 - x^2}$

c. $y = x \sin x$

d. $y = \dfrac{\sec^2 x}{x^2 + 1}$

III. Using Differentials to Approximate Function Values

*To approximate a function value for the function y=f(x), use $f(x+\Delta x) \approx f(x) + dy = f(x) +. f'(x)dx$

Ex. 3: Use linear approximation at x = 2 to estimate f(1.9) given that $f(x) = x^2 - 1$

Ex. 4: Use differentials to approximate $\sqrt{49.8}$.

You Try: Use differentials to approximate $(2.99)^3$.