Antiderivatives & Indefinite Integration (4.1)

February 6th, 2019

I. Antiderivatives

Def. of an Antiderivative: A function F is an antiderivative of f on an interval I if F'(x)=f(x) for all x in I.

*Since the derivative of any constant C is zero, it is impossible to find a single function f for which F is the antiderivative..

Thm. 4.1: Representation of Antiderivatives: If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form G(x)=F(x)+C, for all x in I, where C is a constant.

*Given a function $f(x) = 3x^2$, the function $G(x) = x^3 + C$ represents all possible antiderivatives of f, so it is the general antiderivative of f, where C is called the constant of integration. We would also say that $G(x) = x^3 + C$ is the general solution of the differential equation $G'(x) = 3x^2$. Keep in mind that a differential equation of x and y is an equation that involves x, y, and the derivatives of y.

II. Notation for Antiderivatives

Given a differential equation, $\frac{dy}{dx} = f(x)$ or dy=f(x)dx, finding all of the solutions of this equation is called antidifferentiation, or indefinite integration. The solution is

$$y = \int f(x)\,dx = F(x) + C$$

integral sign — integrand — variable of integration — Constant of integration

**indefinite integral=antiderivative*

III. Basic Integration Rules

*Integration & differentiation are inverses, thus

$$\int F'(x)\,dx = F(x) + C$$

and

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$$

*In addition, all the basic integration rules are mainly just the differentiation rules you've already learned, but in reverse. Also, you can't integrate products and quotients piece by piece, just like with differentiation.

Basic Integration Rules:

(highlight all rules)

Differentiation Formula	Integration Formula
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)dx = k\int f(x)dx$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

Differentiation Formula

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Integration Formula

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Ex. 1: Find the indefinite integral

a. $\int \left(-\frac{3}{x^2} \right) dx$

b. $\int \left(\frac{5}{x^4} - \frac{2}{x^2} + 4x^2 \right) dx$

c. $\int \left(y^5 \sqrt{y} \right) dy$

d. $\int \left(\frac{\cos\theta}{1 - \cos^2\theta} \right) d\theta$

e. $\int \frac{2x^2 - 3x + 5}{2x} dx$

IV. Initial Conditions & Particular Solutions

<u>Ex. 2:</u> Solve each differential equation.

a. $g'(x) = 6x^2, g(0) = -1$

b. $f''(x) = x^2, f'(0) = 6, f(0) = 3$

c. $h'(x) = 4e^x, f(0) = 2$

Extending Particle Motion

Assume x(t) represents a particle's position at time t, v(t) represents its velocity at time t, and a(t) represents its acceleration at time t.

1. Given that v(t)=x'(t), we know that $\int v(t)\,dt = x(t) + C$

2. Given that a(t)=v'(t), we know that $\int a(t)\,dt = v(t) + C$

Ex. 3: An egg has been dropped off the top of a 150 ft building with constant acceleration of -32 ft/sec/sec. How far off the ground is the egg after 2 seconds?