Area (4.2)

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Sigma Notation:

The sum of *n* terms $a_1, a_2, a_3, ..., a_n$ is written as

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

Where *i* is the index of summation, a_i is the *i*th term of the sum, and the upper and lower bounds of summation are *n* and 1.

Thm. 4.2: Summation Formulas:

$$1.\sum_{i=1}^{n} c = cn \qquad 2.\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$3.\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad 4.\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Riemann Sums

Ex. 1: Use 4 rectangles of equal width to approximate the area of the region bounded by $f(x) = 8 - x^2$, the x-axis, and the lines x = 0 and x = 2.

(a) L(5)

(b) R(5)

(c) M(5)

Limit Definition of Area

<u>Definition of the Area of a Region in the Plane:</u> Let *f* be nonnegative on the interval [*a, b*]. The area of the region bounded by the graph of *f*, the x-axis, and the vertical lines x = *a* and x = *b* is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{(b-a)i}{n} \right)\left(\frac{b-a}{n} \right)$$

Ex. 2: Find the area of the region described in each.

(a) Bounded by the graph of $f(x)=x^2$ and the x-axis between x = 0 and x = 1.

(b) Bounded by the graph of $f(x)=4x-1$ and the x-axis between x = 1 and x = 3.