The Fundamental Theorem of Calculus (4.4)

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I. The Fundamental Theorem of Calculus

***Thm.4.11: The Fundamental Theorem of Calculus: If f is continuous on the closed interval [a, b] and F is an antiderivative of f on the interval [a, b], then

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$$\int_a^b f(x)dx = F(b) - F(a)$$

*Note that the Fundamental Theorem only works if f is continuous, whereas a Riemann Sum can be used to find the area under a curve whenever f is defined.

Guidelines for Using the Fundamental Theorem of Calculus:

1. This can be used to evaluate a definite integral if you can find the antiderivative function.
2. The following notation is used:

$$\int_a^b f(x)dx = F(x)\Big]_a^b = F(b) - F(a)$$

i.e., $$\int_2^5 x^2\,dx = \frac{x^3}{3}\Bigg]_2^5 = \frac{5^3}{3} - \frac{2^3}{3} = \frac{125}{3} - \frac{8}{3} = 39$$

3. You can leave out the constant of integration C because

$$\int_a^b f(x)dx = \left[F(x)+C\right]_a^b = \left[F(b)+C\right] - \left[F(a)+C\right] = F(b) - F(a)$$

Ex. 1: Evaluate each definite integral.

a. $\int_{1}^{5} (x^2 - 2x + 3)\,dx$

b. $\int_{1}^{4} 2x\sqrt{x}\,dx$

c. $\int_{\pi/6}^{\pi/3} \csc x \cot x\,dx$

d. $\int_{0}^{4} |3x - 1|\,dx$

e. $\int_{1}^{8} \frac{-2}{x}\,dx$

Ex. 2: Find the area of the region bounded by the graph of $y = 3x^2 + 4x - 2$,the x-axis, and the vertical lines x=1 and x=4.

II. The Mean Value Theorem for Integrals

Thm. 4.12: The Mean Value Theorem for Integrals: If f is continuous on the closed interval [a, b], then there exists a number c in the closed interval [a, b] such that

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$$\int_a^b f(x)dx = f(c)(b-a)$$

III. Average Value of a Function

Def. of the Average Value of a Function on an Interval:
If f is integrable on the closed interval [a, b], then the average value of f on the interval is

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$$\frac{1}{b-a}\int_a^b f(x)dx$$

Ex. 3: Find the average value of $f(x) = 5x^2 - 7$ on the interval [1, 3].

IV. The Second Fundamental Theorem of Calculus

*So far we have used the definite integral as a number, written as $\int_a^b f(x)dx$. We can also write the definite integral as an <u>accumulation function</u> that gives an accumulated area under the curve of f. We write the <u>accumulation function</u> as

$$\int_a^x f(t)dt$$.

Thm. 4.13: The Second Fundamental Theorem of Calculus: If f is continuous on an open interval I containing a, then, for every x in the interval,

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$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Ex 4: Evaluate $\dfrac{d}{dx}\left[\displaystyle\int_{1}^{x} \sqrt[4]{t}\, dt\right]$.

You Try: Evaluate $\dfrac{d}{dx}\left[\displaystyle\int_{9}^{x} te^{3t}\,dt\right]$.

Ex. 5: Evaluate $\frac{d}{dx}\left[\int_{0}^{x^2} \sin\theta^2 \, d\theta\right]$.

You Try: Evaluate $\frac{d}{dx}\left[\int_{-1}^{\sin x}\left(t^2+4t\right)dt\right]$.

Particle Motion

The displacement of a particle from time t=a to t=b is given by $\int_a^b v(t)\,dt$.

The total distance traveled by a particle from time t=a to t=b is given by $\int_a^b |v(t)|\,dt$.

Ex. 6: The velocity of a particle in feet per second is given by the function $v(t) = t^2 - 7t + 6$.

(a) Find the displacement of the particle from 0 to 5 seconds.

(b) Find the distance traveled by the particle from 0 to 5 seconds.