Integration by Substitution (4.5)

February 25th, 2019

I. Pattern Recognition & Change of Variables

Thm. 4.15: Antidifferentiation of a Composite Function: Let g be a function whose range is on the interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C \quad .$$

Using u-substitution, if we let u = g(x), then du = g'(x)dx and $\int f(u)\,du = F(u) + C$.

Guidelines for Making a Change of Variable

1. Choose a substitution $u = g(x)$, usually choosing the inner piece of a composite function.
2. Find $du = g'(x)dx$.
3. Rewrite the integral in terms of u.
4. Find the integral in terms of u.
5. Replace u with $g(x)$.
6. Check by differentiating the result.

Ex. 1: Find each indefinite integral.

a. $\int (5x^2 - 2)^2 (10x)\,dx$

b. $\int (9x^2 + 5)^2 (18x)\,dx$

c. $\int 3\sin 3x\,dx$

d. $\int 4\sec^2 4x\,dx$

e. $\int y(3y^2 - 1)^2\,dy$

f. $\int x(5x^2 + 6)^2\,dx$

g. $\int \sqrt{4x+1}\,dx$

h. $\int \sqrt{9x-2}\,dx$

i. $\int \cos^2 5x \sin 5x\,dx$

Ex. 2: Find each indefinite integral.

a) $\int 3e^{x/2}\,dx$

b) $\int 2xe^{x^2}\,dx$

c) $\int 2e^{x}\left(4-e^{x}\right)^{4}dx$

II. The General Power Rule for Integration

Thm. 4.16: The General Power Rule for Integration: If g is a differentiable function of x, then

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$$\int [g(x)]^n g'(x)dx = \frac{[g(x)]^{n+1}}{n+1} + C, n \neq -1$$

Equivalently, if u = g(x), then

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

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<u>Ex. 3:</u> Find each indefinite integral.

a. $\int 10x(5x^2 - 2)^4 \, dx$

b. $\int (6x^2 - 6)(2x^3 - 6x)^5 \, dx$

c. $\int 4x^3 \sqrt{x^4 + 1} \, dx$

d. $\int \frac{6x}{(3x^2 + 4)^2} \, dx$

Ex. 4: Find $\int (3x^2 + x)^2 \, dx$.

III. Change of Variables for Definite Integrals

Thm. 4.17: Change of Variables for Definite Integrals: If the function u = g(x) has a continuous derivative on the closed interval [a, b] and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Ex. 5: Evaluate each definite integral.

a. $\int_0^2 x(x^2+5)^2\,dx$

b. $\int_{\pi/3}^{\pi} \sin\frac{x}{2}\,dx$

c. $\int_1^2 e^{3x}\,dx$

d. $\int_1^4 \frac{5dx}{3x-1}$

Ex. 6: Evaluate each.

a) $\int_{0}^{2} \frac{x}{\sqrt{4x+1}}\,dx$

b) $\int_{-1}^{2} 2x(3x+4)^4\,dx$

IV. Integration of Even & Odd Functions

<u>Thm. 4.18: Integration of Even & Odd Functions:</u> Let f be integrable on the closed interval [-a, a].

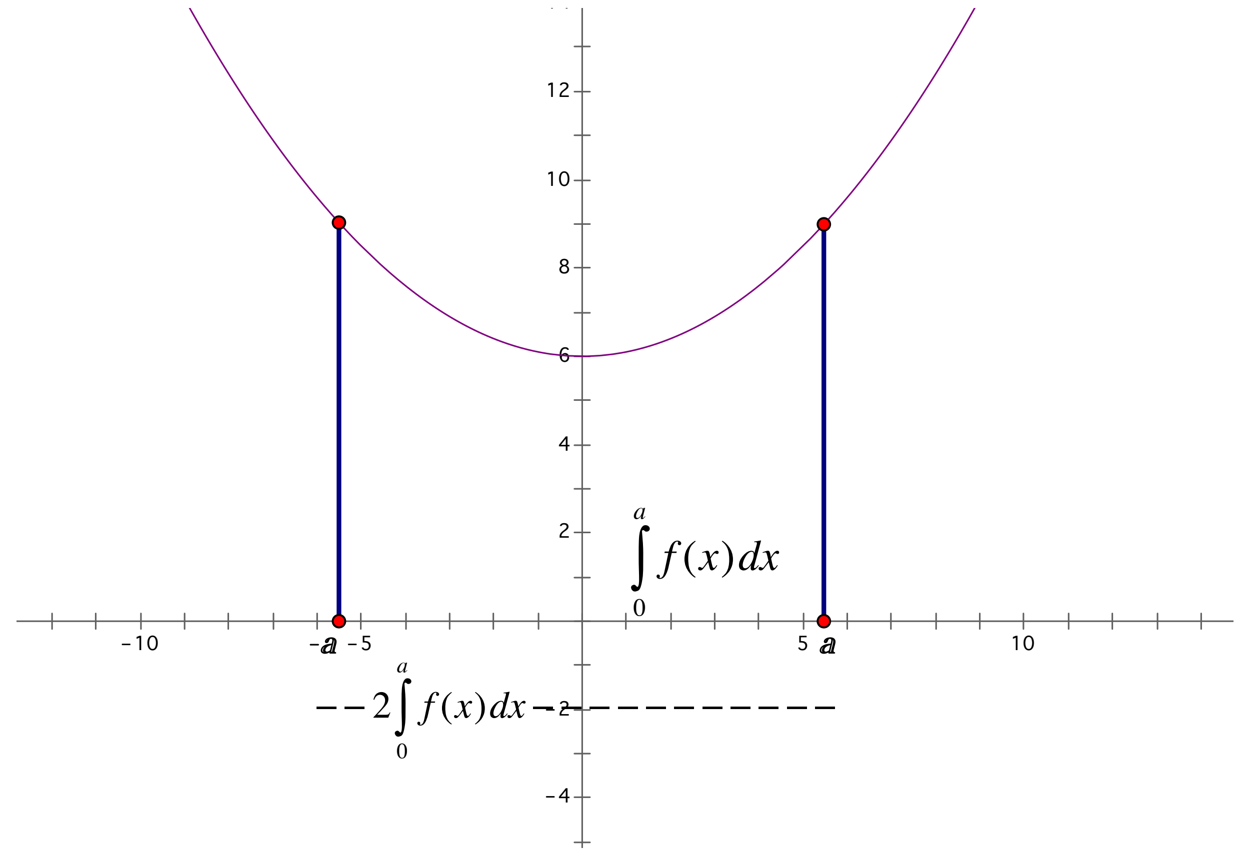
1. If f is an even function, then

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

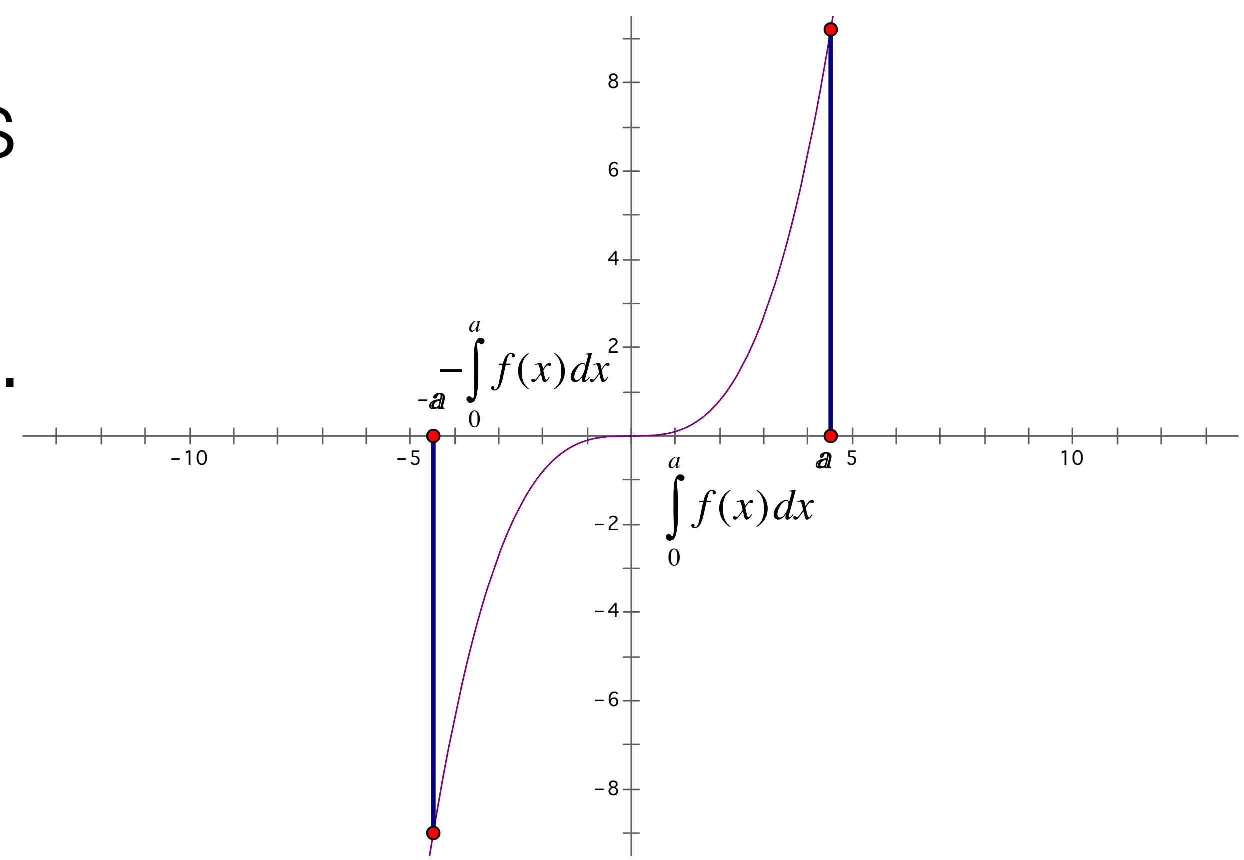
2. If f is an odd function, then

$$\int_{-a}^{a} f(x)\,dx = 0$$

*Recall that even functions, or functions in which f(-x)=f(x), are symmetric about the y-axis.



Odd functions, or functions in which f(-x)=-f(x), are symmetric about the origin.



Ex. 7: Evaluate $\displaystyle\int_{-\pi/2}^{\pi/2} \sin x \cos x \, dx$.