Riemann Sums & Trapezoidal Sums as Estimates for Definite Integrals

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Riemann Sums

*A Riemann Sum is the sum of the areas of a finite number of rectangles that can be used to approximate a definite integral. The Left Riemann Sum uses rectangles whose height is determined by the left endpoint of the rectangle, whereas the Right Riemann Sum uses the right endpoint and the Midpoint Riemann Sum uses the midpoint.

Ex. 1: Approximate $\int_{-2}^{6}(-x^2+36)dx$ by using a

(a) left Riemann sum with 4 equal subintervals

(b) right Riemann sum with 4 equal subintervals

(c) midpoint Riemann sum with 4 equal subintervals

Then, determine which of these gives the largest overestimate of the integral and the smallest underestimate of the integral.

t	0	1	2	4	6	10	14
v(t)	0	20	32	35	30	15	40

Ex. 2: The table above gives the velocity in miles per hour of a car traveling for t hours. Approximate the total distance traveled by the car using a

(a) left Riemann sum

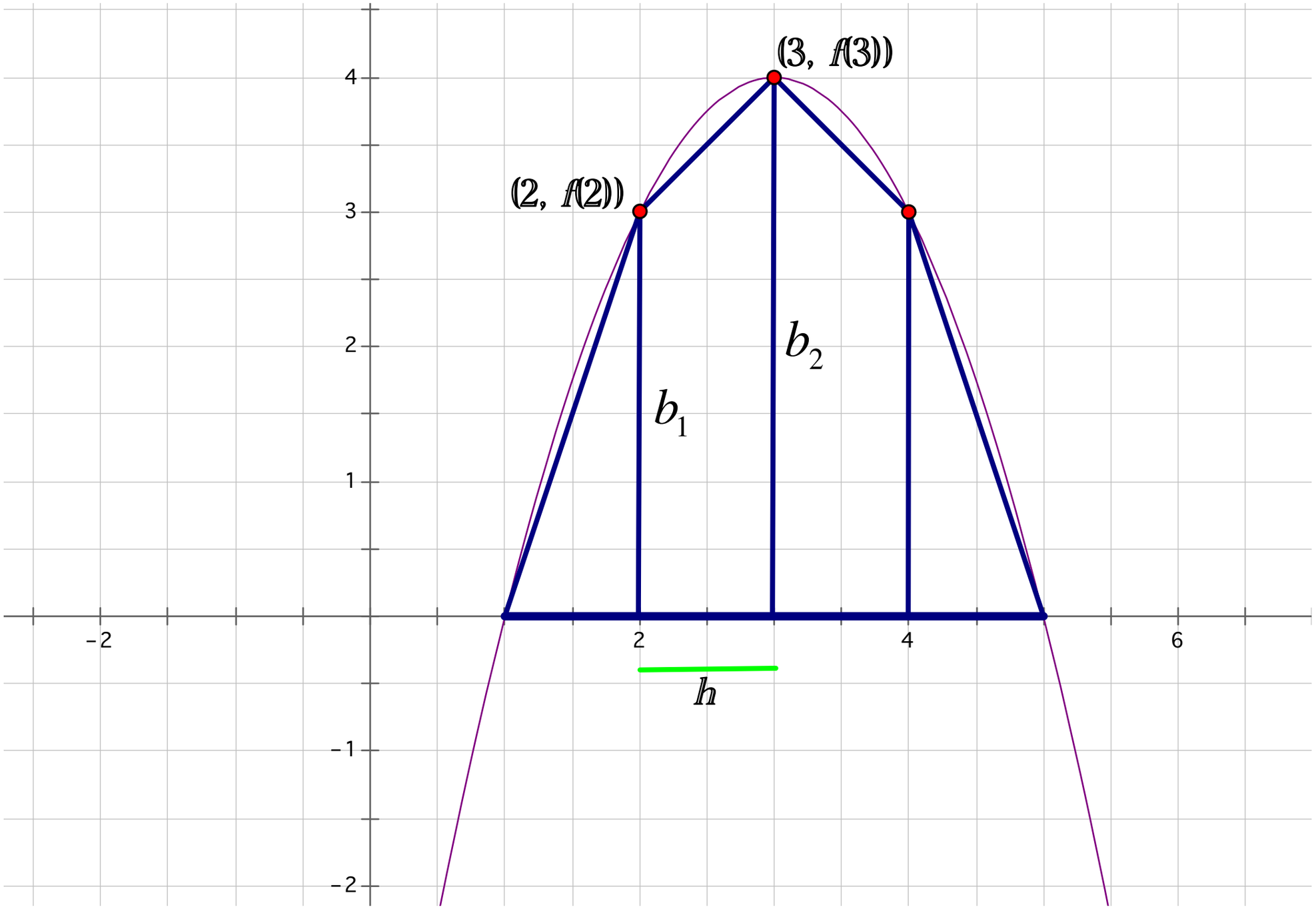
(b) right Riemann sum

(c) midpoint Riemann sum

*A trapezoidal sum uses a finite number of trapezoids (instead of rectangles) to approximate the area under a curve given by a definite integral.

*Recall the area of a trapezoid:

$$Area = \frac{h}{2}(b_1 + b_2)$$



Ex 3: Find the trapezoidal sum for each function over the given interval with 4 subintervals.

a) $f(x) = -x^2 + 2, [0,4]$

b)

x	-1	2	5	9	11
f(x)	-4	-1	2	3	6